
Chapter 3

The Effects of Ground

The ground around and under an antenna is part of the environment in which any actual antenna must operate. Chapter 2 dealt mainly with theoretical antennas in free space, completely removed from the influence of the ground. This chapter is devoted to exploring the interactions between antennas and the ground.

The interactions can be analyzed depending on where they occur relative to two areas surrounding the antenna: the *reactive near field* and the *radiating far field*. You will recall that the reactive near field only exists very close to the antenna itself. In this region the antenna acts as though it were a large lumped-constant inductor or capacitor, where energy is stored but very little is actually radiated. The interaction with the ground in this area creates mutual impedances between the antenna and its environment and these interactions not only modify the feed-point impedance of an antenna, but often increase losses.

In the radiating far field, the presence of ground profoundly influences the radiation pattern of a real antenna. The interaction is different, depending on the antenna's polarization with respect to the ground. For horizontally polarized antennas, the *shape* of the radiated pattern in the elevation plane depends primarily on the antenna's height above ground. For vertically polarized antennas, both the *shape* and the *strength* of the radiated pattern in the elevation plane strongly depend on the nature of the ground itself (its dielectric constant and conductivity at RF), as well as on the height of the antenna above ground.

The Effects of Ground in the Reactive Near Field

FEED-POINT IMPEDANCE VERSUS HEIGHT ABOVE GROUND

Waves radiated from the antenna directly downward reflect vertically from the ground and, in passing the antenna on their upward journey, induce a voltage in it. The magnitude and phase of the current resulting from this induced voltage depends on the height of the antenna above the reflecting surface.

The total current in the antenna consists of two components. The amplitude of the first is determined by the power supplied by the transmitter and the free-space feed-point resistance of the antenna. The second component is induced in the antenna by the wave reflected from the ground. This second component of current, while considerably smaller than the first at most useful antenna heights, is by no means insignificant. At some heights, the two components will be in phase, so the total current is larger than is indicated by the free-space feed-point resistance. At other heights, the two components are out of phase, and the total current is the difference between the two components.

Changing the height of the antenna above ground will change the amount of current flow, assuming that the power input to the antenna is constant. A higher current at the same power input means that the effective resistance of the antenna is lower, and vice versa. In other words, the feed-point resistance of the antenna is affected by the height of the antenna above ground because of mutual coupling between the antenna and the ground beneath it.

The electrical characteristics of the ground affect both the amplitude and the phase of reflected signals. For this reason, the electrical characteristics of the ground under the antenna will have some

effect on the impedance of that antenna, the reflected wave having been influenced by the ground. Different impedance values may be encountered when an antenna is erected at identical heights but over different types of earth.

Fig 1 shows the way in which the radiation resistance of horizontal and vertical half-wave antennas vary with height above ground (in λ , wavelengths). For horizontally polarized half-wave antennas, the differences between the effects of perfect ground and real earth are negligible if the antenna height is greater than 0.2λ . At lower heights, the feed-point resistance over perfect ground decreases rapidly as the antenna is brought closer to a theoretically perfect ground, but this does not occur so rapidly for actual ground. Over real earth, the resistance begins increasing at heights below about 0.08λ . The reason for the increasing resistance at very low heights is that more and more of the reactive (induction) field of the antenna is absorbed by the lossy ground in close proximity.

For a vertically polarized $\lambda/2$ -long dipole, differences between the effects of perfect ground and real earth on the feed-point impedance is negligible, as seen in Fig 1. The theoretical half-wave antennas on which this chart is based are assumed to have infinitely thin conductors.

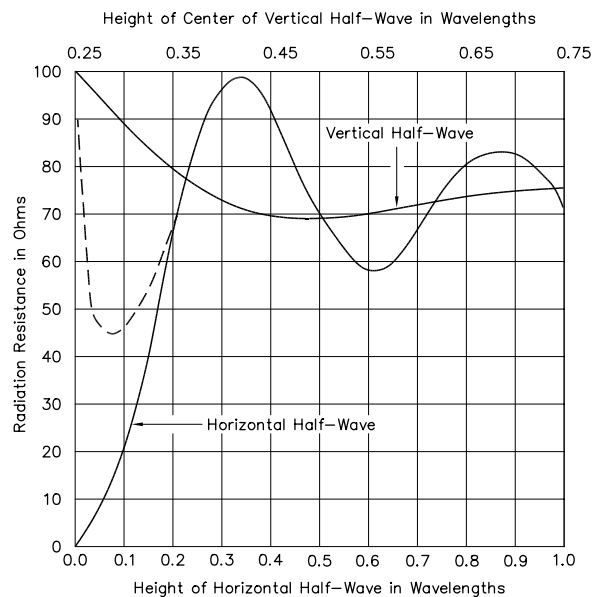


Fig 1—Variation in radiation resistance of vertical and horizontal half-wave antennas at various heights above flat ground. Solid lines are for perfectly conducting ground; the broken line is the radiation resistance of horizontal half-wave antennas at low heights over real ground.

GROUND SYSTEMS FOR VERTICAL MONOPOLES

In this section, we'll look at vertical monopoles, which require some sort of ground system in order to make up for the "missing" second half of the antenna. In [Chapter 2](#) and up to this point in this chapter, the discussion about vertical monopoles has mainly been for antennas where "perfect ground" is available. We have also briefly looked at the ground-plane vertical in free space, where the four ground-plane radials form a built-in "ground" system.

Perfect ground makes a vertical monopole into the functional equivalent of a center-fed dipole, although the feed-point resistance at resonance is half that of the center-fed dipole. But how can we manage to create that elusive "perfect ground" for our real vertical antennas?

Simulating a Perfect Ground in the Reactive Near Field

The effect of a perfectly conducting ground (as far as feed-point resistance and losses are concerned) can be simulated under a real antenna by installing a very large metal screen or mesh, such as poultry netting (chicken wire) or hardware cloth, on or near the surface of the ground. The screen (also called a counterpoise system, especially if it is elevated off the ground) should extend at least a half wavelength in every direction from the antenna. The feed-point resistance of a quarter-wave long, thin vertical radiator over such a ground screen will approach the theoretical value of 36Ω .

Based on the results of a study published in 1937 by Brown, Lewis and Epstein (see [Bibliography](#)), a grounding system consisting of 120 wires, each at least $\lambda/2$ long, extending radially from the base of the antenna and spaced equally around a circle, is also the practical equivalent of perfectly conducting ground for reactive field currents. The wires can either be laid directly on the surface of the ground or buried a few inches below.

Another approach to simulating a perfect ground system is to utilize the ground-plane antenna,

with its four ground-plane radials elevated well above lossy earth. Heights greater than $\lambda/8$ have proven to yield excellent results. See [Chapter 6](#) for more details on practical ground-plane verticals.

For a vertical antenna, a large ground screen, either made of wire mesh or a multitude of radials, or an elevated system of ground-plane radials will reduce ground losses near the antenna. This is because the screen conductors are solidly bonded to each other and the resistance is much lower than that of the lossy, low-conductivity earth itself. If the ground screen or elevated ground plane were not present, RF currents would be forced to flow through the lossy, low-conductivity earth to return to the base of the radiator. The ground screen or elevated ground plane in effect shield ground-return currents from the lossy earth.

Less-Than-Ideal Ground Systems

Now, what happens when something less than an ideal ground screen is used as the ground plane for a vertical monopole? You will recall from [Chapter 2](#) that an ideal ground-plane antenna in free space requires only four radials as a ground counterpoise. Thus, the four-radial ground plane antenna in free space represents one limit in the range of possibilities for a ground system, while a perfect ground screen represents the other limit. Real antenna systems over real ground represent intermediate points in this continuum of ground configurations.

A great deal of mystery and lack of information seems to surround the vertical antenna ground system. In the case of ground-mounted vertical antennas, many general statements such as “the more radials the better” and “lots of short radials are better than a few long ones” have served as rules of thumb to some, but many questions as to relative performance differences and optimum number for a given length remain unanswered. Most of these questions boil down to one: namely, how many radials, and how long, should be used in a vertical antenna installation?

A ground system with 120 $\lambda/2$ radials is not very practical for many amateur installations, which often must contend with limited space for putting together such an ideal system. Unfortunately, ground-return loss resistance increases rapidly when the number of radials is reduced. At least 15 radials should be used if at all possible. Experimental measurements show that with this number, the loss resistance is such as to decrease the antenna efficiency to about 50% if the monopole vertical length is $\lambda/4$.

As the number of radials is reduced, the vertical radiator length required for optimum results with a particular number of radials also decreases—in other words, if only a small number of radials can be used with a shortened vertical radiator, there is no point in extending them out $\lambda/2$. This comes about because the reactive near field of a short vertical radiator extends out radially less than that for a full-sized $\lambda/4$ vertical. With 15 radials, for example, a radiator length of $\lambda/8$ is sufficient. With as few as two radials the length is almost unimportant, but the efficiency of a $\lambda/4$ antenna with such a grounding system is only about 25%. (It is considerably lower with shorter antennas.)

In general, a large number of radials (even though some or all of them must be short) is preferable to a few long radials for a vertical antenna mounted on the ground. The conductor size is relatively unimportant; #12 to #28 copper wire is suitable. The measurement of the actual ground-loss resistance at the operating frequency is difficult. The power loss in the ground depends on the current concentration near the base of the antenna, and this depends on the antenna height. Typical values for small radial systems (15 or less) have been measured to be from about 5 to 30 Ω , for antenna heights from $\lambda/16$ to $\lambda/4$. The impedance seen at the feed point of the antenna is the sum of the loss and the radiation resistance.

Table 1 summarizes these conclusions. John Stanley, K4ERO, first presented this material in December 1976 *QST*. One source of information on ground-system design is *Radio Broadcast Ground Systems* (see the [Bibliography](#) at the end of this chapter). Most of the data presented in Table 1 is taken from that source, or derived from the interpolation of data contained therein.

Table 1 gives numbers of radials and a corresponding optimum radial length for each case. Using radials considerably longer than suggested for a given number or using a lot more radials than suggested for a given length, while not adverse to performance, does not yield significant improvement

Table 1
Optimum Ground-System Configurations

	<i>Configuration Designation</i>					
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
Number of radials	16	24	36	60	90	120
Length of each radial in wavelengths	0.1	0.125	0.15	0.2	0.25	0.4
Spacing of radials in degrees	22.5	15	10	6	4	3
Total length of radial wire installed, in wavelengths	1.6	3	5.4	12	22.5	48
Power loss in dB at low angles with a quarter-wave radiating element	3	2	1.5	1	0.5	0*
Feed-point impedance in ohms with a quarter-wave radiating element	52	46	43	40	37	35

Note: Configuration designations are indicated only for text reference.
 *Reference. The loss of this configuration is negligible compared to a perfectly conducting ground.

either. That would represent a nonoptimum use of wire and construction time. Each suggested configuration represents an optimum relationship between length and number for a fixed amount of total. The loss figures in Table 1 are calculated for a quarter-wave radiating element. A very rough approximation of loss when using shorter antennas can be obtained by doubling the loss in dB each time the antenna height is halved. For longer antennas the losses decrease, approaching 2 dB for configuration A of Table 1 for a half-wave radiator. Longer antennas yield correspondingly better performance.

The table is based on average ground conductivity. Variation of the loss values shown can be considerable, especially for configurations using fewer radials. Those building antennas over dry, sandy or rocky ground should expect more loss. On the other hand, higher than average soil conductivity and wet soils would make the “compromise” configurations (those with the fewest radials) even more attractive.

When antennas are combined into arrays, either of parasitic or all-driven types, mutual impedances lower the radiation resistance of the elements, drastically increasing the effects of ground loss. For instance, an antenna with a 52-Ω feed-point impedance and 10 Ω of ground-loss resistance will have an efficiency of approximately 83%. An array of two similar antennas in a driven array with the same ground loss may have an efficiency of 70% or less. Special precautions must be taken in such cases to achieve satisfactory operation. Generally speaking, a wide-spaced broadside array presents little problem, but a close-spaced end-fire array should be avoided for transmission, unless the lower loss configurations are used or other precautions taken. [Chapter 8](#) covers the subject of vertical arrays in great detail.

In cases where directivity is desirable or real estate limitations dictates, longer, more closely spaced radials can be installed in one direction, and shorter, more widely spaced in another. Multiband ground systems can be designed using different optimum configurations for different bands. Usually it is most convenient to start at the lowest frequency with fewer radials and add more short radials for better performance on the higher bands.

There is nothing sacred about the exact details of the configurations given, and slight changes in the number of radials and lengths will not cause serious problems. Thus, a configuration with 32 or 40 radials of 0.14 λ or 0.16 λ will work as well as configuration C shown in the table.

If less than 90 radials are contemplated, there is no need to make them a quarter wavelength long. This differs rather dramatically from the case of a ground plane antenna, where four λ/4 resonant radials are installed above ground. For the ground-mounted antenna, four λ/4 radials are far from optimum. Because the radials of a ground-mounted vertical are actually on, if not slightly below the surface, they are coupled by capacitance or conduction to the ground, and thus resonance effects are not important. The basic function of radials is to provide a low-loss return path for ground currents. The

reason that short radials are sufficient when few are used is that at the perimeter of the circle to which the ground system extends, the few wires are so spread apart that most of the return currents are already in the ground between the wires rather than in the wires themselves. As more wires are added, the spaces between them are reduced and longer length helps to provide a path for currents still farther out.

Radio Broadcast Ground Systems states, “Experiments show that the ground system consisting of only 15 radial wires need not be more than 0.1 wavelength long, while the system consisting of 113 radials is still effective out to 0.5 wavelength.” Many graphs in that publication confirm this statement. This is not to say that these two systems will perform equally well; they most certainly will not. However, if 0.1λ is as long as the radials can be, there is little point in using more than 15 of them.

The antenna designer should (1) study the cost of various radial configurations versus the gain of each; (2) compare alternative means of improving transmitted signal and their cost (more power, etc); (3) consider increasing the physical antenna height (the electrical length) of the vertical radiator, instead of improving the ground system; and (4) use multielement arrays for directivity and gain, observing the necessary precautions related to mutual impedances discussed in [Chapter 8](#).

The Effect of Ground in the Far Field

The properties of the ground in the far field of an antenna are very important, especially for a vertically polarized antenna. Even if the ground system for a vertical has been optimized to reduce ground-return losses in the reactive near field to an insignificant level, the electrical properties of the ground may still diminish far-field performance to lower levels than “perfect-ground” analyses might lead you to expect. The key is that ground reflections from horizontally and vertically polarized waves behave very differently.

Reflections in General

Over flat ground, both horizontally or vertically polarized downgoing waves launched from an antenna into the far field strike the surface and are reflected by a process very similar to that by which light waves are reflected from a mirror. As is the case with light waves, the angle of reflection is the same as the angle of incidence, so a wave striking the surface at an angle of, say, 15° is reflected upward from the surface at 15° .

The reflected waves combine with direct waves (those radiated at angles above the horizon) in various ways. Some of the factors that influence this combining process are the height of the antenna, its length, the electrical characteristics of the ground, and as mentioned above, the polarization of the wave. At some elevation angles above the horizon the direct and reflected waves are exactly in phase—that is, the maximum field strengths of both waves are reached at the same time at the same point in space, and the directions of the fields are the same. In such a case, the resultant field strength for that angle is simply the sum of the direct and reflected fields. (This represents a theoretical increase in field strength of 6 dB over the free-space pattern at these angles.)

At other elevation angles the two waves are completely out of phase—that is, the field intensities are equal at the same instant and the directions are opposite. At still other angles, the resultant field will have intermediate values. Thus, the effect of the ground is to increase radiation intensity at some elevation angles and to decrease it at others. When you plot the results as an elevation pattern, you will see *lobes* and *nulls*, as described in [Chapter 2](#).

The concept of an image antenna is often useful to show the effect of reflection. As [Fig 2](#) shows, the reflected ray has the same path length (AD equals BD) that it would if it originated at a virtual second antenna with the same characteristics as the real antenna, but situated below the ground just as far as the actual antenna is above it.

Now, if we look at the antenna and its image over perfect ground from a remote point on the surface of the ground, we will see that the currents in a horizontally polarized antenna and its image are flowing in opposite directions, or in other words, are 180° out of phase. But the currents in a vertically polarized antenna and its image are flowing in the *same* direction—they are *in* phase. This 180° phase

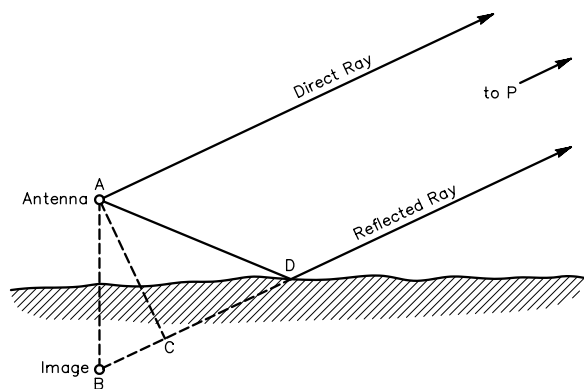


Fig 2—At any distant point, P, the field strength will be the vector sum of the direct ray and the reflected ray. The reflected ray travels farther than the direct ray by the distance BC, where the reflected ray is considered to originate at the “image” antenna.

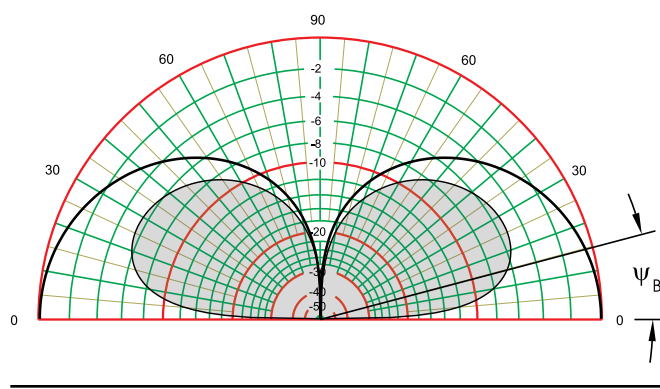


Fig 3—Vertical-plane radiation pattern for a ground-mounted quarter-wave vertical. The solid line is the pattern for perfect earth. The shaded pattern shows how the response is modified over average earth ($k = 13$, $G = 0.005$ S/m) at 14 MHz. ψ is the pseudo-Brewster angle (PBA), in this case 14.8° .

difference between the vertically and horizontally polarized reflections off ground is what makes the combinations with direct waves behave so very differently.

FAR-FIELD GROUND REFLECTIONS AND THE VERTICAL ANTENNA

A vertical’s azimuthal directivity is omnidirectional. A $\lambda/2$ vertical over ideal earth has the elevation-plane radiation pattern shown by the solid line in **Fig 3**. Over real earth, however, the pattern looks more like the shaded one in the same diagram. In this case, the low-angle radiation that might be hoped for because of the perfect-ground performance is not realized.

Now look at **Fig 4A**, which compares the computed elevation-angle response for two half-wave dipoles at 14 MHz. One is oriented horizontally over ground at a height of $\lambda/2$ and the other is oriented vertically, with its center just over $\lambda/4$ high (so that the bottom end of the wire doesn’t actually touch the ground). The ground is “average” in dielectric constant and conductivity. At a 15° elevation angle, the horizontally polarized dipole has almost 7 dB more gain than its vertical brother. Contrast **Fig 4A** to the comparison in **Fig 4B**, where the peak gain of a vertically polarized half-wave dipole over seawater, which is virtually perfect for RF reflections, is quite comparable with the horizontal dipole’s response at 15° , and exceeds the horizontally polarized antenna dramatically below 15° elevation.

To understand why the desired low-angle radiation is not delivered over real earth, examine **Fig 5A**. Radiation from each antenna segment reaches a point P in space by two paths; one directly from the antenna, path AP, and the other by reflection from the earth, path AGP. (Note that P is so far away that the slight difference in angles is insignificant—for practical purposes the waves are parallel to each other at point P.)

If the earth were a perfectly conducting surface, there would be no phase shift of the vertically polarized wave upon reflection at point G. The two waves would add together with some phase difference because of the different path lengths. This difference in path lengths of the two waves is why the free-space radiation pattern differs from the pattern of the same antenna over ground. Now consider a point P that is close to the horizon, as in **Fig 5B**. The path lengths AP and AGP are almost the same, so the magnitudes of the two waves add together, producing a maximum at zero angle of radiation. The arrows on the waves point both ways since the process works similarly for transmitting and receiving.

With real earth, however, the reflected wave undergoes a change in both *amplitude* and *phase* in the reflection process. Indeed, at a low enough elevation angle, the phase of the reflected wave will actually change by 180° and its magnitude will then subtract from that of the direct wave. At a zero

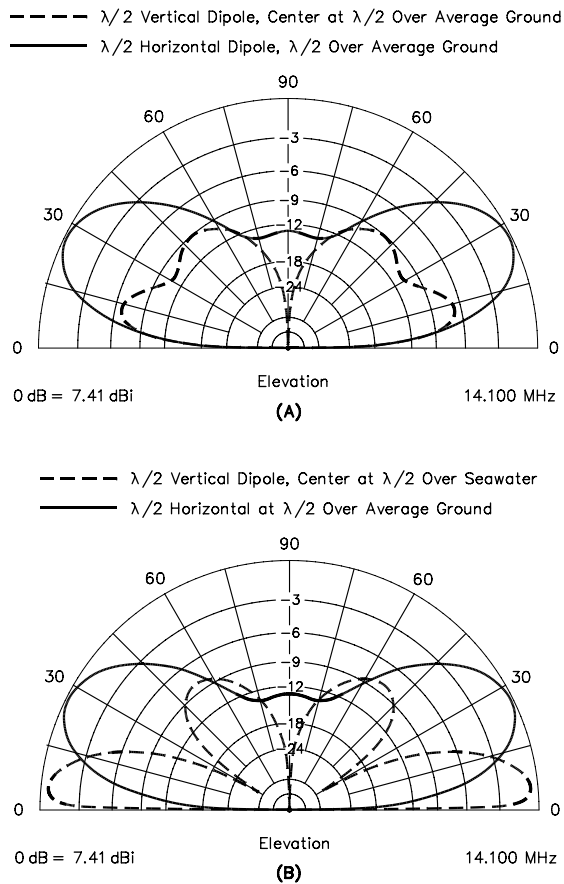


Fig 4—At A, comparison of horizontal and vertical $\lambda/2$ dipoles over average ground. Average ground has conductivity of 5 mS/m and dielectric constant of 13. Center of each antenna is $\lambda/2$ over ground. Horizontal antenna is much less affected by far-field ground losses compared with its vertical counterpart. At B, comparison of 20-meter $\lambda/4$ vertical dipole raised $\lambda/2$ over seawater with $\lambda/2$ horizontal dipole, $\lambda/2$ over average ground. Seawater is great for verticals!

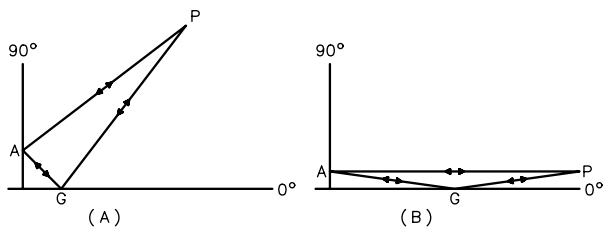


Fig 5—The direct wave and the reflected wave combine at point P to form the pattern (P is very far from the antenna). At A the two paths AP and AGP differ appreciably in length, while at B these two path lengths are nearly equal.

takeoff angle, it will be almost equal in amplitude, but 180° out of phase with the direct wave. Complete cancellation will result in a null, inhibiting any radiation or reception at 0° .

THE PSEUDO-BREWSTER ANGLE AND THE VERTICAL ANTENNA

Much of the material presented here regarding pseudo-Brewster angle was prepared by Charles J. Michaels, W7XC, and first appeared in July 1987 *QST*, with additional information in *The ARRL Antenna Compendium, Vol 3*. (See the [Bibliography](#) at the end of this chapter.)

Most fishermen have noticed that when the sun is low, its light is reflected from the water's surface as glare, obscuring the underwater view. When the sun is high, however, the sunlight penetrates the water and it is possible to see objects below the surface of the water. The angle at which this transition takes place is known as the *Brewster angle*, named for the Scottish physicist, Sir David Brewster (1781-1868).

A similar situation exists in the case of vertically polarized antennas; the RF energy behaves as the sunlight in the optical system, and the earth under the antenna acts as the water. The *pseudo-Brewster angle* (PBA) is the angle at which the reflected wave is 90° out of phase with respect to the direct wave. "Pseudo" is used here because the RF effect is similar to the optical effect from which the term gets its name. Below this angle, the reflected wave is between 90° and 180° out of phase with the direct wave, so some degree of cancellation takes place. The largest amount of cancellation occurs near 0° , and steadily less cancellation occurs as the PBA is approached from below.

The factors that determine the PBA for a particular location *are not related to the antenna itself, but to the ground around it*. The first of these factors is earth conductivity, G , which is a measure of the ability of the soil to conduct electricity. Conductivity is the inverse of resistance. The second factor is the dielectric constant, k , which is a unitless quantity that corresponds to the capacitive effect of the earth. For both of these quantities, the higher the number, the better the ground (for vertical antenna purposes). The third factor determining the PBA for a given location is the frequency of operation. The PBA increases with increasing frequency, all other conditions being equal. [Table 2](#) gives typical values of conductivity and

Table 2**Conductivities and Dielectric Constants for Common Types of Earth**

<i>Surface Type</i>	<i>Dielectric Constant</i>	<i>Conductivity (S/m)</i>	<i>Relative Quality</i>
Fresh water	80	0.001	
Salt water	81	5.0	
Pastoral, low hills, rich soil, typ Dallas, TX, to Lincoln, NE areas	20	0.0303	Very good
Pastoral, low hills, rich soil, typ OH and IL	14	0.01	
Flat country, marshy, densely wooded, typ LA near Mississippi River	12	0.0075	
Pastoral, medium hills and forestation, typ MD, PA, NY (exclusive of mountains and coastline)	13	0.006	
Pastoral, medium hills and forestation, heavy clay soil, typ central VA	13	0.005	Average
Rocky soil, steep hills, typ mountainous	12-14	0.002	Poor
Sandy, dry, flat, coastal	10	0.002	
Cities, industrial areas	5	0.001	Very Poor
Cities, heavy industrial areas, high buildings	3	0.001	Extremely poor

dielectric constant for different types of soil. The map of **Fig 6** shows the approximate conductivity values for different areas in the continental United States.

As the frequency is increased, the role of the dielectric constant in determining the PBA becomes more significant. **Table 3** shows how the PBA varies with changes in ground conductivity, dielectric constant and frequency. The table shows trends in PBA dependency on ground constants and frequency. The constants chosen are not necessarily typical of any geographical area; they are just examples.

At angles below the PBA, the reflected vertically polarized wave subtracts from the direct wave, causing the radiation intensity to fall off rapidly. Similarly, above the PBA, the reflected wave adds to the direct wave, and the radiated pattern approaches the perfect-earth pattern. **Fig 3** shows the PBA, usually labeled ψ_B .

When plotting vertical-antenna radiation patterns over real earth, the reflected wave from an antenna segment is multiplied by a factor called the *vertical reflection coefficient*, and the product is then added vectorially to the direct wave to get the resultant. The reflection coefficient consists of an attenuation factor, A , and a phase angle, ϕ , and is usually expressed as $A\angle\phi$. (ϕ is always a negative angle, because the earth acts as a lossy capacitor in this situation.) The following equation can be used to calculate the reflection coefficient for vertically polarized waves, for earth of given conductivity and dielectric constant at any frequency and elevation angle (also called the wave angle in many texts).

$$A_{\text{Vert}} \angle\phi = \frac{k' \sin \Psi - \sqrt{k'' - \cos^2 \Psi}}{k' \sin \Psi + \sqrt{k'' - \cos^2 \Psi}} \quad (\text{Eq 1})$$

where

$A_{\text{Vert}} \angle\phi$ = vertical reflection coefficient

Ψ = elevation angle

$$k' = k - j \left(\frac{1.8 \times 10^4 \times G}{f} \right)$$

k = dielectric constant of earth (k for air = 1)

G = conductivity of earth in S/m

Table 3
Pseudo-Brewster Angle Variation with Frequency, Dielectric Constant, and Conductivity

Frequency, (MHz)	Dielectric constant	Conductivity, (S/m)	PBA, (degrees)
7	20	0.0303	6.4
	13	0.005	13.3
	13	0.002	15.0
	5	0.001	23.2
	3	0.001	27.8
14	20	0.0303	8.6
	13	0.005	14.8
	13	0.002	15.4
	5	0.001	23.8
	3	0.001	29.5
21	20	0.0303	10.0
	13	0.005	15.2
	13	0.002	15.4
	5	0.001	24.0
	3	0.001	29.8

f = frequency in MHz
 j = complex operator ($\sqrt{-1}$)

Solving this equation for several points indicates what effect the earth has on vertically polarized signals at a particular location for a given frequency range. Fig 7 shows the reflection coefficient as a function of elevation angle at 21 MHz over average earth ($G = 0.005$ S/m, $k = 13$). Note that as the phase curve, ψ , passes through 90° , the attenuation curve, A , passes through a minimum at the same wave angle, ψ . This is the PBA. At this angle, the reflected wave is not only at a phase angle of 90° with respect to the direct wave, but is so low in amplitude that it does not aid the direct wave by a significant amount. In the case illustrated in Fig 7 this elevation angle is about 15° .

Variations in PBA with Earth Quality

From Eq 1, it is quite a task to search for either the 90° phase point or the attenuation curve

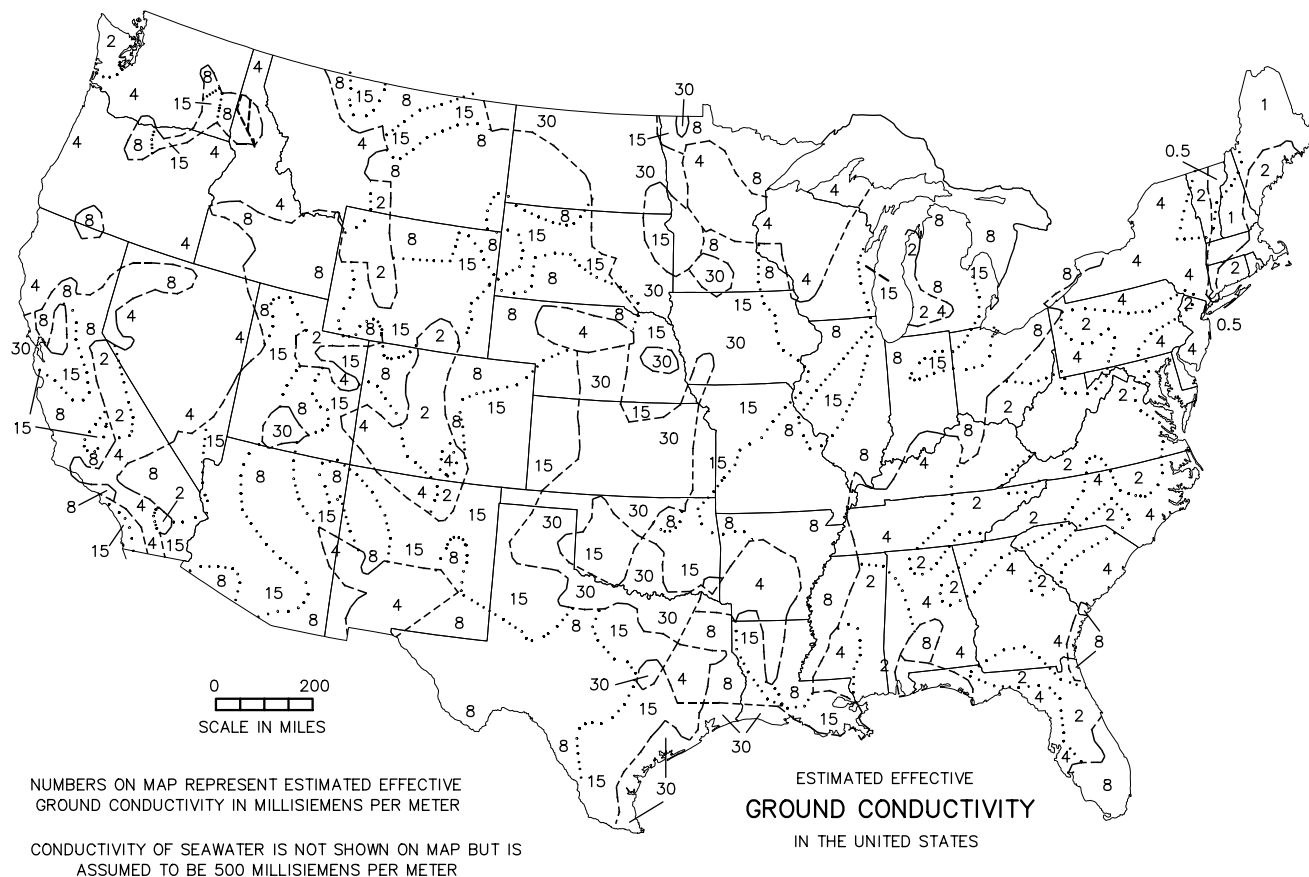


Fig 6—Typical average soil conductivities for the continental United States. Numeric values indicate conductivities in millisiemens per meter (mS/m), where 1.0 mS/m = 0.001 S/m.

minimum for a wide variety of earth conditions. Instead, the PBA can be calculated directly from the following equation.

$$\Psi_B = \sqrt{\frac{k - 1 + \sqrt{(x^2 + k^2)^2 (k - 1)^2 + x^2 [(x^2 + k^2)^2 - 1]}}{(x^2 + k^2)^2 - 1}} \quad (\text{Eq 2})$$

where

$$x = \frac{1.8 \times 10^4 \times G}{f}$$

k, G and f are as defined for Eq 1.

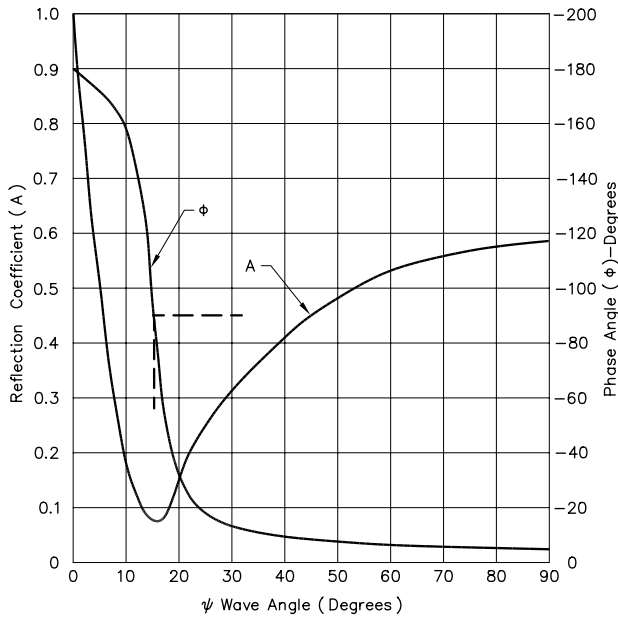


Fig 7—Reflection coefficient for vertically polarized waves. A and φ are magnitude and angle for wave angles ψ. This case is for average earth, (k = 13, G = 0.005 S/m), at 21 MHz.

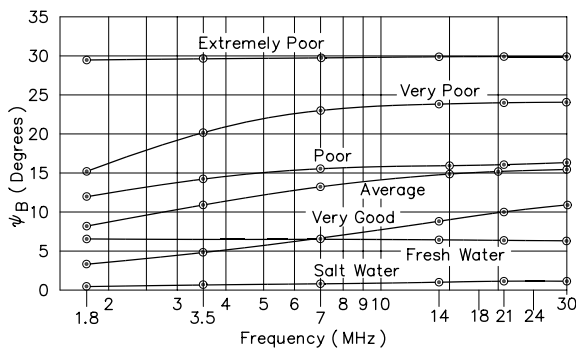


Fig 8—Pseudo-Brewster angle (ψ) for various qualities of earth over the 1.8 to 30-MHz frequency range. Note that the frequency scale is logarithmic. The constants used for each curve are given in Table 2.

Fig 8 shows curves calculated using Eq 2 for several different earth conditions, at frequencies between 1.8 and 30 MHz. As expected, poorer earths yield higher PBAs. Unfortunately, at the higher frequencies (where low-angle radiation is most important for DX work), the PBAs are highest. The PBA is the same for both transmitting and receiving.

Relating PBA to Location and Frequency

Table 2 lists the physical descriptions of various kinds of earth with their respective conductivities and dielectric constants, as mentioned earlier. Note that in general, the dielectric constants and conductivities are higher for better earths. This enables the labeling of the earth characteristics as extremely poor, very poor, poor, average, very good, and so on, without the complications that would result from treating the two parameters independently.

Fresh water and salt water are special cases; in spite of high resistivity, the fresh-water PBA is 6.4°, and is nearly independent of frequency below 30 MHz. Salt water, because of its extremely high conductivity, has a PBA that never exceeds 1° in this frequency range. The extremely low conductivity listed for cities (last case) in **Table 2** results more from the clutter of surrounding buildings and other obstructions than any actual earth characteristic. The PBA at any location can be found for a given frequency from the curves in **Fig 8**.

FLAT-GROUND REFLECTIONS AND HORIZONTALLY POLARIZED WAVES

The situation for horizontal antennas is different from that of verticals. **Fig 9** shows the reflection coefficient for horizontally polarized

waves over average earth at 21 MHz. Note that in this case, the phase-angle departure from 0° never gets very large, and the attenuation factor that causes the most loss for high-angle signals approaches unity for low angles. Attenuation increases with progressively poorer earth types. In calculating the broadside radiation pattern of a horizontal $\lambda/2$ dipole, the perfect-earth image current, equal to the true antenna current but 180° out of phase with it) is multiplied by the horizontal reflection coefficient given by Eq 3 below. The product is then added vectorially to the direct wave to get the resultant at that elevation angle. The reflection coefficient for horizontally polarized waves can be calculated using the following equation.

$$A_{\text{Horiz}} \angle \phi = \frac{\sqrt{k' - \cos^2 \Psi} - \sin \Psi}{\sqrt{k' - \cos^2 \Psi} + \sin \Psi} \quad (\text{Eq 3})$$

where

$A_{\text{Horiz}} \angle \phi$ = horizontal reflection coefficient

Ψ = elevation angle

$$k' = k - j \left(\frac{1.8 \times 10^4 \times G}{f} \right)$$

k = dielectric constant of earth

G = conductivity of earth in S/m

f = frequency in MHz

j = complex operator ($\sqrt{-1}$)

For a horizontal antenna near the earth, the resultant pattern is a modification of the free-space pattern of the antenna. **Fig 10** shows how this modification takes place for a horizontal $\lambda/2$ antenna over a perfectly conducting flat surface. The patterns at the left show the relative radiation when one views the antenna from the side; those at the right show the radiation pattern looking at the end of the

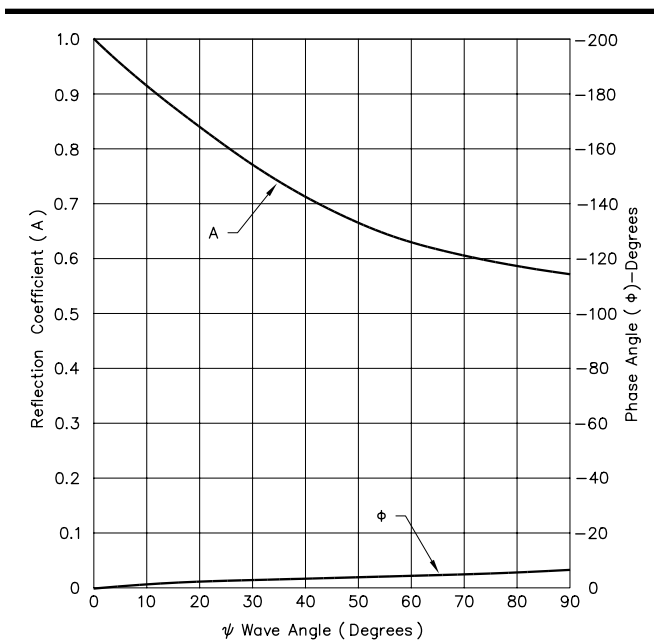


Fig 9—Reflection coefficient for horizontally polarized waves (magnitude A at angle ϕ), at 21 MHz over average earth ($k = 13$, $G = 0.005$ S/m).

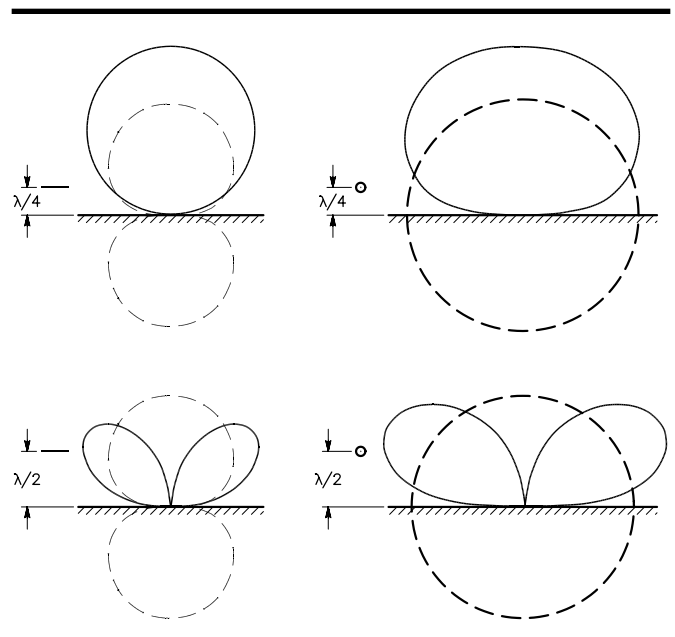


Fig 10—Effect of the ground on the radiation from a horizontal half-wave antenna, for heights of one-fourth and one-half wavelength. Broken lines show what the pattern would be if there were no reflection from the ground (free space).

antenna. Changing the height above ground from $\lambda/4$ to $\lambda/2$ makes a significant difference in the high-angle radiation, moving the main lobe down lower.

Note that for an antenna height of $\lambda/2$ (Fig 10, bottom), the out-of-phase reflection from a perfectly conducting surface creates a null in the pattern at the zenith (90° elevation angle). Over real earth, however, a “filling in” of this null occurs because of ground losses that prevent perfect reflection of high-angle radiation.

At a 0° elevation angle, horizontally polarized antennas also demonstrate a null, because out-of-phase reflection cancels the direct wave. As the elevation angle departs from 0° , however, there is a slight filling-in effect so that over other-than-perfect earth, radiation at lower angles is enhanced compared to a vertical. A horizontal antenna will often outperform a vertical for low-angle DX work, particularly over lossy types of earth at the higher frequencies.

Reflection coefficients for vertically and horizontally polarized radiation differ considerably at most angles above ground, as can be seen by comparison of Figs 7 and 8. (Both sets of curves were plotted for the same ground constants and at the same frequency, so they may be compared directly.) This is because, as mentioned earlier, the image of a horizontally polarized antenna is out of phase with the antenna itself, and the image of a vertical antenna is in phase with the actual radiator.

The result is that the phase shifts and reflection magnitudes vary greatly at different angles for horizontal and vertical polarization. The magnitude of the reflection coefficient for vertically polarized waves is greatest (near unity) at very low angles, and the phase angle is close to 180° . As mentioned earlier, this cancels nearly all radiation at very low angles. For the same range of angles, the magnitude of the reflection coefficient for horizontally polarized waves is also near unity, but the phase angle is near 0° for the specific conditions shown in Figs 7 and 9. This causes reinforcement of low-angle horizontally polarized waves. At some relatively high angle, the reflection coefficients for horizontally and vertically polarized waves are equal in magnitude and phase. At this angle (approximately 81° for the example case), the effect of ground reflection on vertically and horizontally polarized signals will be exactly the same.

DEPTH OF RF CURRENT PENETRATION

When considering earth characteristics, questions about depth of RF current penetration often arise. For instance, if a given location consists of a 6-foot layer of soil overlying a highly resistive rock strata, which material dominates? The answer depends on the frequency, the soil and rock dielectric constants, and their respective conductivities. The following equation can be used to calculate the current density at any depth.

$$e^{-pd} = \frac{\text{Current Density at Depth D}}{\text{Current Density at Surface}} \quad (\text{Eq 4})$$

where

$$p = \left(\frac{X \times B}{2} \times \left(\sqrt{1 + \frac{G^2 \times 10^{-4}}{B^2}} - 1 \right) \right)^{1/2}$$

d = depth of penetration in cm

e = natural logarithm base (2.718)

X = $0.008 \times \pi^2 \times f$

B = $5.56 \times 10^{-7} \times k \times f$

k = dielectric constant of earth

f = frequency in MHz

G = conductivity of earth in S/m

After some manipulation of this equation, it can be used to calculate the depth at which the current density is some fraction of that at the surface. The depth at which the current density is 37% ($1/e$) of that at the surface (often referred to as *skin depth*) is the depth at which the current density would be

zero if it were distributed uniformly instead of exponentially. (This $1/e$ factor appears in many physical situations. For instance, a capacitor charges to within $1/e$ of full charge within one RC time constant.) At this depth, since the power loss is proportional to the square of the current, approximately 91% of the total power loss has occurred, as has most of the phase shift, and current flow below this level is negligible.

Fig 11 shows the solutions to Eq 4 over the 1.8 to 30-MHz frequency range for various types of earth. For example, in very good earth, substantial RF currents flow down to about 3.3 feet at 14 MHz. This depth goes to 13 feet in average earth and as far as 40 feet in very poor earth. Thus, if the overlying soil is rich, moist loam, the underlying rock strata is of little concern. However, if the soil is only average, the underlying rock may constitute a major consideration in determining the PBA and the depth to which the RF current will penetrate.

The depth in fresh water is about 156 feet and is nearly independent of frequency in the amateur bands below 30 MHz. In salt water, the depth is about seven inches at 1.8 MHz and decreases rather steadily to about two inches at 30 MHz. Dissolved minerals in moist earth increase its conductivity.

The depth-of-penetration curves in Fig 11 illustrate a noteworthy phenomenon. While skin effect confines RF current flow close to the surface of a conductor, the earth is so lossy that RF current penetrates to much greater depths than in most other media. The depth of RF current penetration is a function of frequency as well as earth type. Thus, the only cases in which most of the current flows near the surface are with very highly conductive media (such as salt water), and at frequencies above 30 MHz.

DIRECTIVE PATTERNS OVER REAL GROUND

As explained in Chapter 2, because antenna radiation patterns are three-dimensional, it is helpful in understanding their operation to use a form of representation showing the vertical directional characteristic for different heights. It is possible to show selected vertical-plane patterns oriented in various directions with respect to the antenna axis. In the case of the horizontal half-wave dipole, a plane running in a direction along the axis and another broadside to the antenna will give a good deal of information.

The effect of reflection from the ground can be expressed as a separate *pattern factor*, given in decibels. For any given elevation angle, adding this factor algebraically to the value for that angle from

the free-space pattern for that antenna gives the resultant radiation value at that angle. The limiting conditions are those represented by the direct ray and the reflected ray being exactly in phase and exactly out of phase, when both, assuming there are no ground losses, have equal amplitudes. Thus, the resultant field strength at a distant point may be either 6 dB greater than the free-space pattern (twice the field strength), or zero, in the limiting cases.

Horizontally Polarized Antennas

The way in which pattern factors vary with height for horizontal antennas over flat earth is shown graphically in the plots of **Fig 12**. The solid-line plots are based on perfectly conducting ground, while the shaded plots are based on typical real-earth conditions. These patterns apply to horizontal antennas of any length. While these graphs are, in fact, radiation patterns of horizontal single-wire antennas (dipoles) as viewed from

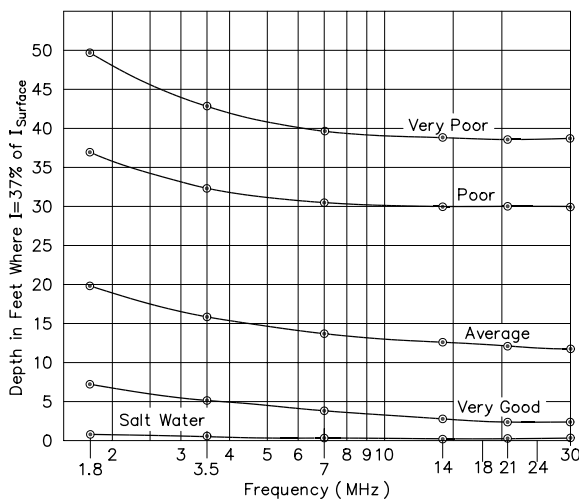


Fig 11—Depths at which the current density is 37% of that at the surface for different qualities of earth over the 1.8 to 30-MHz frequency range. The depth for fresh water, not plotted, is 156 feet and almost independent of frequency below 30 MHz. See text and **Table 2** for ground constants.

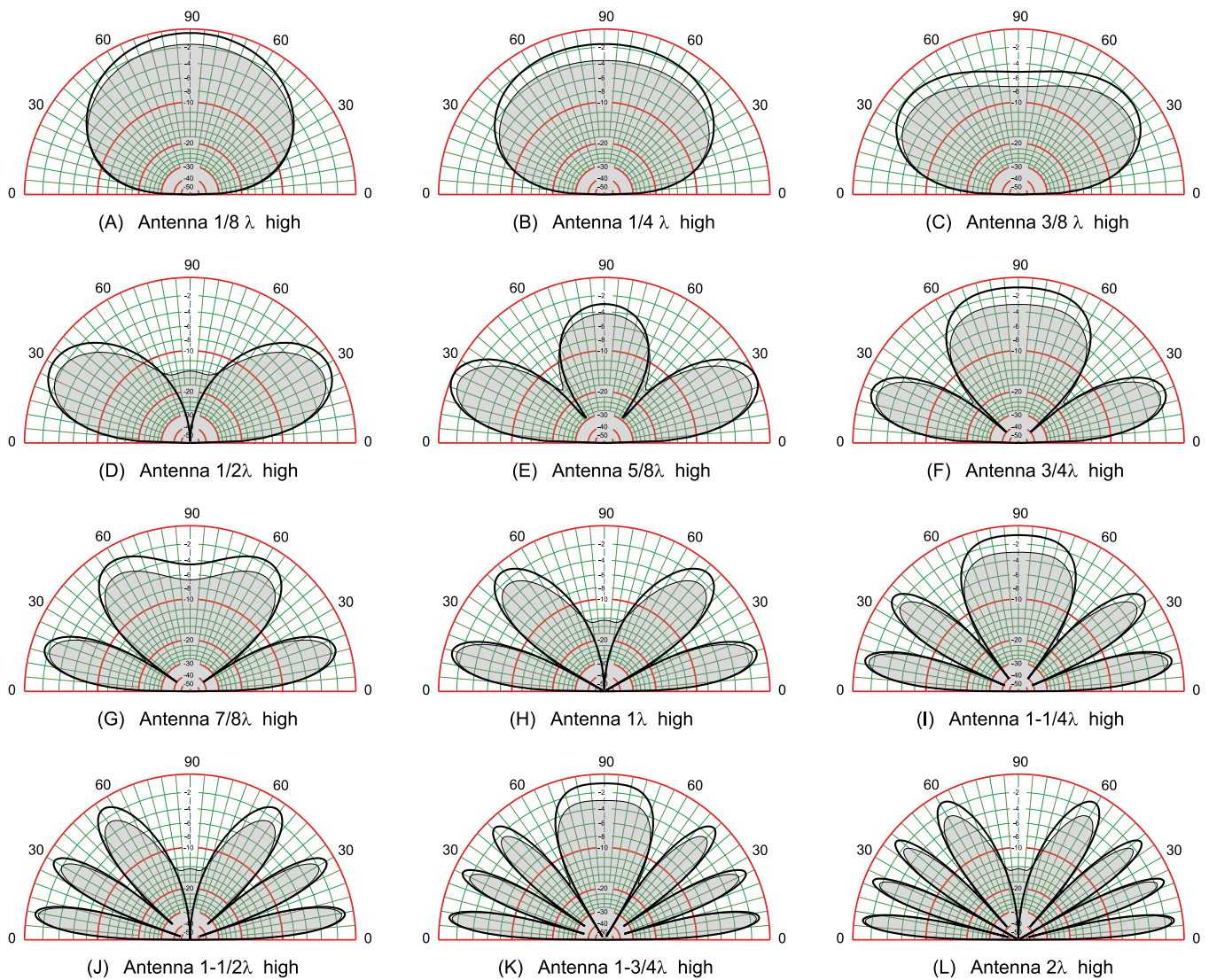


Fig 12—Reflection factors for horizontal antennas at various heights above flat ground. The solid-line curves are the perfect-earth patterns (broadside to the antenna wire); the shaded curves represent the effects of average earth ($k = 13$, $G = 0.005 \text{ S/m}$) at 14 MHz. Add 7 dB to values shown for absolute gain in dBd referenced to dipole in free space, or 9.15 dB for gain in dBi. For example, peak gain over perfect earth at $3/8 \lambda$ height is 7 dBd (or 9.15 dBi) at 25° elevation.

the axis of the wire, it must be remembered that the plots merely represent pattern factors.

Vertical radiation patterns in the directions off the ends of a horizontal half-wave dipole are shown in **Fig 13** for various antenna heights. These patterns are scaled so they may be compared directly to those for the appropriate heights in Fig 12. Note that the perfect-earth patterns in Figs 13A and 12B are the same as those in the upper part of Fig 10. Note also that the perfect-earth patterns of Figs 13B and 12D are the same as those in the lower section of Fig 10. The reduction in field strength off the ends of the wire at the lower angles, as compared with the broadside field strength, is quite apparent. It is also clear from Fig 13 that, at some heights, the high-angle radiation off the ends is nearly as great as the broadside radiation, making the antenna essentially an omnidirectional radiator.

In vertical planes making some intermediate angle between 0° and 90° with the wire axis, the pattern will have a shape intermediate between the broadside and end-on patterns. By visualizing a smooth transition from the end-on pattern to the broadside pattern as the horizontal angle is varied from 0° to 90° , a fairly good mental picture of the actual solid pattern may be formed. An example is shown in **Fig 14**. At A, the vertical pattern of a half-wave dipole at a height of $\lambda/2$ is shown through a

plane 45° away from the favored direction of the antenna. At B and C, the vertical pattern of the same antenna is shown at heights of $3\lambda/4$ and 1λ (through the same 45° off-axis plane). These patterns are scaled so they may be compared directly with the broadside and end-on patterns for the same antenna (at the appropriate heights) in Figs 12 and 13.

The curves presented in Fig 15 are useful for determining heights of horizontal antennas that give either maximum or minimum reinforcement at any desired wave angle. For instance, if you want to place an antenna at a height so that it will have a null at 30° , the antenna should be placed where a broken line crosses the 30° line on the horizontal scale. There are two heights (up to 2λ) that will yield this null angle: 1λ and 2λ .

As a second example, you may want to have the ground reflection give maximum reinforcement of the direct ray from a horizontal antenna at a 20° elevation angle. The antenna height should be 0.75λ . The same height will give a null at 42° and a second lobe at 90° .

Fig 15 is also useful for visualizing the vertical pattern of a horizontal antenna. For example, if an antenna is erected at 1.25λ , it will have major lobes (solid-line crossings) at 12° and 37° , as well as at 90° (the zenith). The nulls in this pattern (dashed-line crossings) will appear at 24° and 53° . By using Fig 15

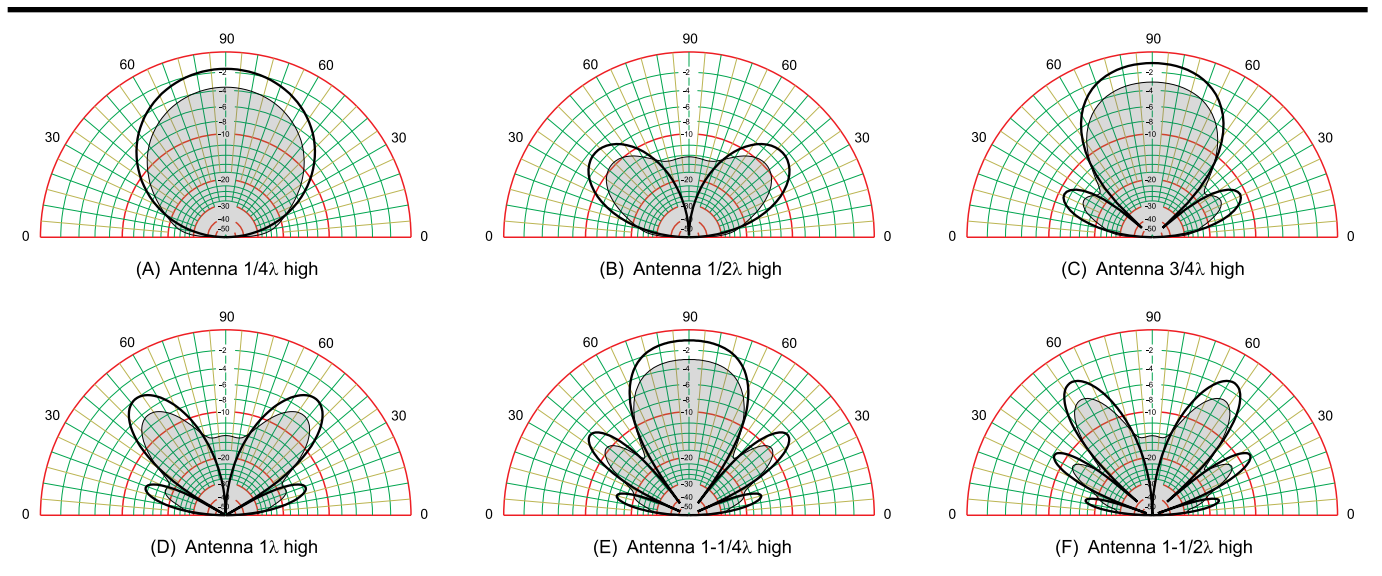


Fig 13—Vertical-plane radiation patterns of horizontal half-wave antennas off the ends of the antenna wire. The solid-line curves are the flat, perfect-earth patterns, and the shaded curves represent the effects of average flat earth ($k = 13$, $G = 0.005$ S/m) at 14 MHz. The 0-dB reference in each plot corresponds to the peak of the main lobe in the favored direction of the antenna (the maximum gain). Add 7 dB to values shown for absolute gain in dBd referenced to dipole in free space, or 9.15 dB for gain in dBi.

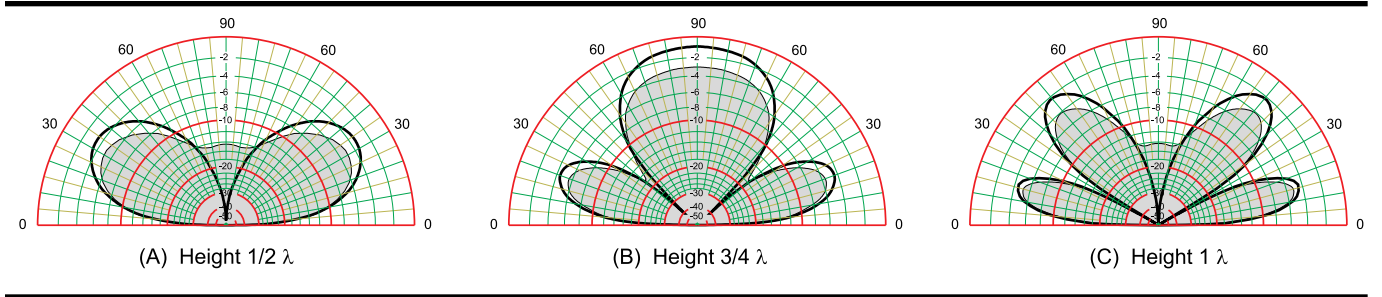


Fig 14—Vertical-plane radiation patterns of half-wave horizontal antennas at 45° from the antenna wire over flat ground. The solid-line and shaded curves represent the same conditions as in Figs 12 and 13. These patterns are scaled so they may be compared directly with those of Figs 12 and 13.

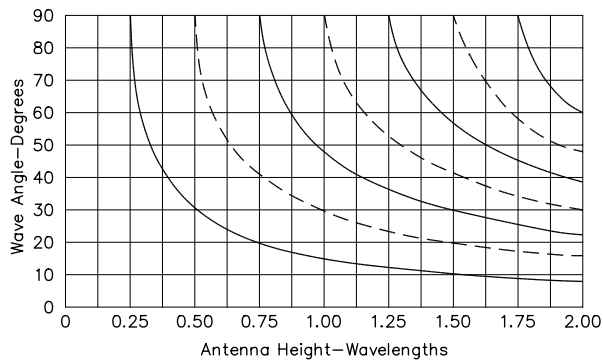


Fig 15—Angles at which nulls and maxima (factor = 6 dB) in the ground reflection factor appear for antenna heights up to two wavelengths over flat ground. The solid lines are maxima, dashed lines nulls, for all horizontal antennas. See text for examples. Values may also be determined from the trigonometric relationship $\theta = \arcsin(A/4h)$, where θ is the wave angle and h is the antenna height in wavelengths. For the first maximum, A has a value of 1; for the first null A has a value of 2, for the second maximum 3, for the second null 4, and so on.

along with wave-angle information contained in Chapter 23, it is possible to calculate the antenna height that will best suit your needs, remembering that this is for flat-earth terrain.

Vertically Polarized Antennas

In the case of a vertical $\lambda/2$ dipole or a ground-plane antenna, the horizontal directional pattern is simply a circle at any elevation angle (although the actual field strength will vary, at the different elevation angles, with the height above ground). Hence, one vertical pattern is sufficient to give complete information (for a given antenna height) about the antenna in any direction with respect to the wire. A series of such patterns for various heights is given in **Fig 16**. The three-dimensional radiation pattern in each case is formed by rotating the plane pattern about the zenith axis of the graph.

The solid-line curves represent the radiation patterns of the $\lambda/2$ vertical dipole at different feed-point heights over perfectly conducting ground. The shaded curves show the patterns produced by the same antennas at the same heights over average ground ($G = 0.005 \text{ S/m}$, $k = 13$) at 14 MHz. The PBA in this case is 14.8° .

In short, far-field losses for vertically polarized antennas are highly dependent on the conductivity and dielectric constant of the earth around the antenna, extending far beyond the ends of any radials used to complete the ground return for the near field.

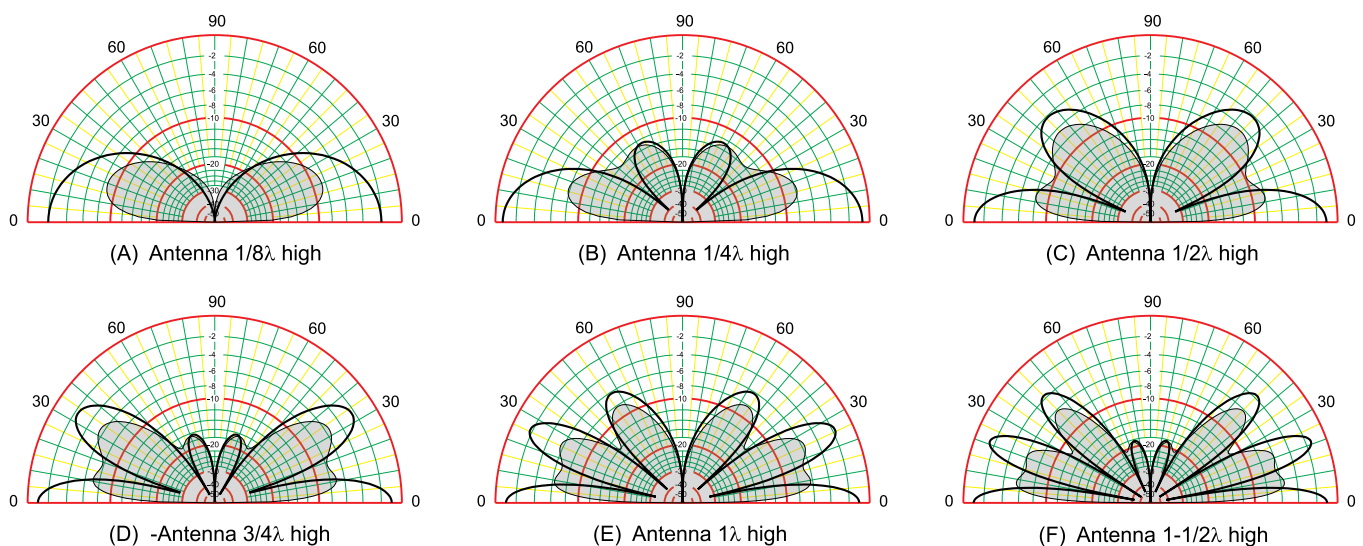


Fig 16—Vertical-plane radiation patterns of a groundplane antenna above flat ground. The height is that of the ground plane, which consists of four radials in a horizontal plane. Solid lines are perfect-earth patterns; shaded curves show the effects of real earth. The patterns are scaled—that is, they may be directly compared to the solid-line ones for comparison of losses at any wave angle. These patterns were calculated for average ground ($k = 13$, $G = 5 \text{ mS/m}$) at 14 MHz. The PBA for these conditions is 14.8° . Add 6 dB to values shown for absolute gain in dBd over dipole in free space.

Putting more radials out around the antenna may well decrease ground-return losses in the reactive near field for a vertical monopole, but will not increase radiation at low elevation launch angles in the far field, unless the radials can extend perhaps 100 wavelengths in all directions! Aside from moving to the fabled “salt water swamp on a high hill,” there is very little that someone can do to change the character of the ground that affects the far-field pattern of a real vertical. Classical texts on verticals often show elevation patterns computed over an “infinitely wide, infinitely conducting ground plane.” Real ground, with finite conductivity and less-than-perfect dielectric constant, can severely curtail the low-angle radiation at which verticals are supposed to excel.

While real verticals over real ground are not a sure-fire method to achieve low-angle radiation, cost versus performance and ease of installation are still attributes that can highly recommend verticals to knowledgeable builders. Practical installations for 160 and 80 meters rarely allow amateurs to put up a horizontal antenna high enough to radiate effectively at low elevation angles. After all, a half-wave on 1.8 MHz is 273 feet high, and even at such a lofty height the peak radiation would be at a 30° elevation angle.

The Effects of Irregular Local Terrain in the Far Field

The following material is condensed and updated from an article by R. Dean Straw, N6BV, in July 1995 *QEX* magazine. The *YT* program, standing for “Yagi Terrain Analysis,” and supporting data files are included on the CD-ROM.

Choosing a QTH for DXing

The subject of how to choose a QTH for working DX has fascinated hams since the beginning of amateur operations. No doubt, Marconi probably spent a lot of time wandering around Newfoundland looking for a great radio QTH before making the first transatlantic transmission. Putting together a high-performance HF station for contesting or DXing has always followed some pretty simple rules. First, you need the perfect QTH, preferably on a rural mountain top or at least on top of a hill. Even better yet, you need a mountaintop surrounded by seawater! Then, after you have found your dream QTH, you put up the biggest antennas you possibly can, on the highest towers you can afford. Then you work all sorts of DX—sunspots willing, of course.

The only trouble with this straightforward formula for success is that it doesn't always work. Hams fortunate enough to be located on mountaintops with really spectacular drop-offs often find that their highest antennas don't do very well, especially on 15 or 10 meters, but often even on 20 meters. When they compare their signals with nearby locals in the flatlands, they sometimes (but not always) come out on the losing end, especially when sunspot activity is high.

On the other hand, when the sunspots drop into the cellar, the high antennas on the mountaintop are usually the ones crunching the pileups—but again, not always. So, the really ambitious contest aficionados, the guys with lots of resources and infinite enthusiasm, have resorted to putting up antennas at all possible heights, on a multitude of towers.

There is a more scientific way to figure out where and how high to put your antennas to optimize your signal during all parts of the 11-year solar cycle. We advocate a *system approach* to HF station design, in which you need to know the following:

1. The range of elevation angles necessary to get from point A to point B
2. The elevation patterns for various types and configurations of antennas
3. The effect of local terrain on elevation patterns for horizontally polarized antennas.

WHAT IS THE RANGE OF ELEVATION ANGLES NEEDED?

Until 1994, *The ARRL Antenna Book* contained only a limited amount of information about the elevation angles needed for communication throughout the world. In the 1974 edition, Table 1-1 in the Wave Propagation chapter was captioned: “Measured vertical angles of arrival of signals from

England at receiving location in New Jersey.”

What the caption didn't say was that Table 1-1 was derived from measurements made during 1934 by Bell Labs. The highest frequency data seemed pretty shaky, considering that 1934 was the low point of Cycle 17. Neither was this data applicable to any other path, other than the one from New Jersey to England. Nonetheless, many amateurs located throughout the US tried to use the sparse information in Table 1-1 as the only rational data they had for determining how high to mount their antennas. (If they lived on hills, they made estimates on the effect of the terrain, assuming that the hill was adequately represented by a long, unbroken slope. More on this later.)

In 1993 ARRL HQ embarked on a major project to tabulate the range of elevation angles from all regions of the US to important DX QTHs around the world. This was accomplished by running many thousands of computations using the *IONCAP* computer program. *IONCAP* has been under development for more than 25 years by various agencies of the US government and is considered the standard of comparison for propagation programs by many agencies, including the Voice of America, Radio Free Europe, and more than 100 foreign governments throughout the world. *IONCAP* is a real pain in the neck to use, but it is the standard of comparison.

The calculations were done for all levels of solar activity, for all months of the year, and for all 24 hours of the day. The results were gathered into some very large databases, from which special custom-written software extracted detailed statistics. The results appeared in summary form in Tables 4 through 13 printed in the chapter “Radio Wave Propagation,” Chapter 23, of the 17th Edition and in more detail on the diskette included with that book. (This book, the 18th Edition, contains even more statistical data, for more areas of the world, on the accompanying diskette.)

Fig 17 reproduces Fig 28 from Chapter 23. This depicts the full range of elevation angles for the 20-meter path from Newington, Connecticut, to all of Europe. This is for all openings, in all months, over the entire 11-year solar cycle. The most likely elevation angle occurs between 10° to 12° for about 42% of the times when the band is open. There is a secondary peak between 4° to 6°, occurring for about 29% of the time the band is open.

In Fig 17, the statistical angle information is also overlaid with the elevation responses for three different antenna configurations, all mounted over flat ground. The stack of four 4-element Yagis at 120, 90, 60 and 30 feet best covers the whole range of necessary elevation angles among the three systems shown, with the best single antenna arguably being the 90-foot high Yagi.

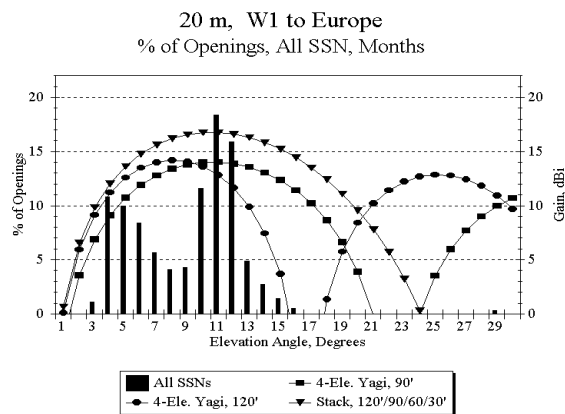


Fig 17—Graph showing 20-meter percentage of all openings from New England to Europe versus elevation angles, together with overlay of elevation patterns over flat ground for three 20-meter antenna systems. The most statistically likely angle at which the band will be open is 11°, although at any particular hour, day, month and year, the actual angle may well be different.

Now, we must emphasize that these are *statistical entities*— in other words, just because 11° is the “statistically most likely angle” for the 20-meter path from New England to Europe doesn't mean that the band will be open at 11° at any particular hour, on a particular day, in a particular month, in any particular year. In fact, however, experience agrees with the *IONCAP* computations: the 20-meter path to Europe from New England usually opens at a low angle in the morning hours, rising to about 11° during the afternoon, when the signals remain strongest throughout the afternoon until the evening.

Now see **Fig 18**. Just because 9° is the statistically most prevalent angle (occurring some 22% of the time) from Seattle to Europe on 20 meters, this doesn't mean that the actual angle *at any particular moment in time* might not be 10°, or even 2°. The statistics for W7 to Europe say that 9° is

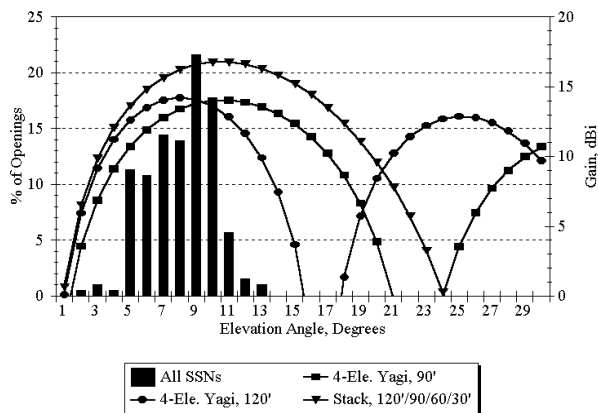


Fig 18—Graph showing 20-meter percentage of all openings, this time from Seattle, WA, to Europe, together with overlay of elevation patterns over flat ground for three 20-meter antenna systems. The statistically most likely angle on this path is 9°, occurring about 22% of the time when the band is actually open. Higher antennas predominate on this path.

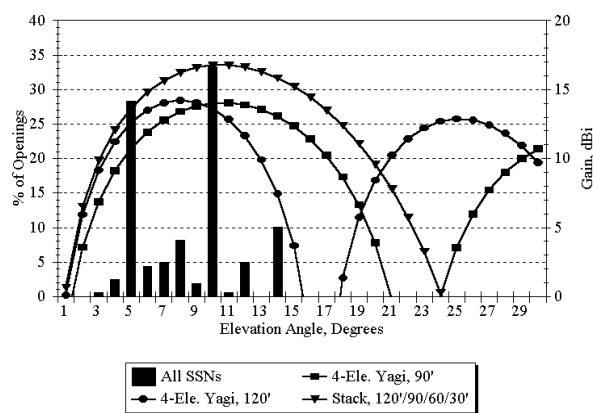


Fig 19—Graph showing 20-meter percentage of all openings from Chicago to Southern Africa, together with overlay of elevation patterns over flat ground for three 20-meter antenna systems. On this long-distance path, higher antennas are most effective.

the most likely angle, but 20-meter signals from Europe arrive at angles ranging from 1° to 13°. If you design an antenna system to cover all possible angles needed to talk to Europe from Seattle (or from Seattle to Europe) on 20 meters, you would need to cover the full range from 1° to 13° equally well.

Similarly, if you wish to cover the full range of elevation angles from Chicago to Southern Africa on 15 meters, you would need to cover 1° to 14°, even though the most statistically likely signals arrive at 10°, for 34% of the time when the band is open for that path. See **Fig 19**.

DRAWBACKS OF COMPUTER MODELS FOR ANTENNAS OVER REAL TERRAIN

Modern general-purpose antenna modeling programs such as *NEC* or *MININEC* (or their commercially upgraded equivalents, such as *NEC/Wires* or *EZNEC*) can accurately model almost any type of antenna commonly used by radio amateurs. In addition, there are specialized programs specifically designed to model Yagis efficiently, such as *YO*, *YA* (*Yagi Analyzer*, included on the diskette with this book) or *YagiMax*. These programs however are all unable to model antennas accurately over anything other than *purely flat ground*.

While both *NEC* and *MININEC* can simulate irregular ground terrain, they do so in a decidedly crude manner, employing step-like concentric rings of height around an antenna. The documentation for *NEC* and *MININEC* both clearly state that diffraction off these “steps” is not modeled. Common experience among serious modelers is that the warnings in the manuals are well worth heeding!

Although analysis and even optimization of antenna designs can be done using free-space or flat-earth ground models, it is *diffraction* that makes the real world a very, very complicated place indeed. This should be clarified—diffraction is hard, even tortuous, to analyze properly, but it makes analysis of real world results far more believable than a flat-world reflection model does.

RAY-TRACING OVER UNEVEN LOCAL TERRAIN

The Raytracing Technique

First, let’s look at a simple raytracing procedure involving only horizontally polarized reflections, with no diffractions. From a specified height on the tower, an antenna shoots “rays” (just as though they were bullets) in 0.25° increments from +35° above the horizon to -35° below the horizon. Each ray is traced over the foreground terrain to see if it hits the ground at any point on its travels in the

direction of interest. If it does hit the ground, the ray is reflected following the classical “law of reflection.” That is, the outgoing angle equals the incoming angle, reflected through the normal to the slope of the surface. Once the rays exit into the ionosphere, the individual contributions are vector-summed to create the overall far-field elevation pattern.

The next step in terrain modeling involves adding diffractions as well as reflections. At the Dayton antenna forum in 1994, Jim Breakall, WA3FET, gave a fascinating and tantalizing lecture on the effect of foreground terrain. Later Breakall, Dick Adler, K3CXZ, Joel Young and a group of other researchers published an extremely interesting paper entitled “The Modeling and Measurement of HF Antenna Skywave Radiation Patterns in Irregular Terrain” in the July 1994 *IEEE Transactions on Antennas and Propagation*. They described in rather general terms the modifications they made to the *NEC-BSC* program. They showed how the addition of a ray-tracing reflection and diffraction model to the simplistic stair-stepped reflection model in regular *NEC* gave far more realistic results. For validation, they compared actual pattern measurements made on a site in Utah (with an overflying helicopter) to computed patterns made using the modified *NEC* software. However, because the work was funded by the US Navy, the software was, and still is, a military secret.

Thumbnail History of the Uniform Theory of Diffraction

It is instructive to look briefly at the history of how “Geometric Optics” (GO) evolved (and still continues to evolve) into the “Uniform Theory of Diffraction” (UTD). The following is summarized from the historical overview in one book found to be particularly useful and comprehensive on the subject of UTD: *Introduction to the Uniform Geometrical Theory of Diffraction*, by McNamara, Pistorius, and Malherbe.

Many years before the time of Christ, the ancient Greeks studied optics. Euclid is credited with deriving the law of reflection about 300 BC. Other Greeks, such as Ptolemy, were also fascinated with optical phenomena. In the 1600s, a Dutchman named Snell finally figured out the law of refraction, resulting in *Snell’s law*. By the early 1800s, the basic world of classical optics was pretty well described from a mathematic point of view, based on the work of a number of individuals.

As its name implies, classical geometric optical theory deals strictly with geometric shapes. Of course, the importance of geometry in optics shouldn’t be minimized—after all, we wouldn’t have eyeglasses without geometric optics. Mathematical analysis of shapes utilizes a methodology that traces the paths of straight-line *rays* of light. (Note that the paths of rays can also be likened to the straight-line paths of particles.) In classical geometric optics, however, there is no mention of three important quantities: phase, intensity and polarization. Indeed, without phase, intensity or polarization, there is no way to deal properly with the phenomenon of *interference*, or its cousin, *diffraction*. These phenomena require theories that deal with *waves* rather than rays.

Wave theory has also been around for a long time, although not as long as geometry. Workers like Hooke and Grimaldi had recorded their observations of interference and diffraction in the mid 1600s. Huygens had used elements of wave theory in the late 1600s to help explain refraction. By the late 1800s, the work of Lord Rayleigh, Sommerfeld, Fresnel, Maxwell and many others led to the full mathematic characterization of all electromagnetic phenomena, light included.

Unfortunately, ray theory doesn’t work for many problems, at least ray theory in the classical optical form. The real world is a lot more jagged, pointy and fuzzy in shape than can be described in a totally rigorous mathematic fashion. Some properties of the real world are most easily explained on the micro level using electrons and protons as conceptual objects, while other macro phenomena (like resonance, for example) are more easily explained in terms of waves. To get a handle on a typical real-world physical situation, a combination of classical ray theory and wave theory was needed.

The breakthrough in the combination of classical geometric optics and wave concepts came from J. B. Keller of Bell Labs in 1953, although he published his work in the early 1960s. In the very simplest of terms, Keller introduced the notion that shooting a ray at a diffraction “wedge” causes wave interference at the tip, with an infinite number of diffracted waves emanating from the diffraction point. Each diffracted wave can be considered to be a point source radiator at the place of generation,

the diffraction point. Thereafter, the paths of individual waves can be traced as though they were individual classical optic rays again. What Keller came up with was a reasonable mathematical description of what happens at the tip of the diffraction wedge.

Fig 20 is a picture of a simple diffraction wedge, with an incoming ray launched at an angle of α_r , referenced to the horizon, impinging on it. The diffraction wedge here is considered to be perfectly conducting, and hence impenetrable by the ray. The wedge generates an infinite number of diffracted waves, going in all directions not blocked by the wedge itself. The amplitudes and phases of the diffracted waves are determined by the interaction at the wedge tip, and this in turn is governed by the various angles associated with the wedge. Shown in Fig 20 are the included angle α of the wedge, the angle ϕ' of the incoming ray (referenced to the incoming surface of the wedge), and the observed angle ϕ of one of the outgoing diffracted waves, also referenced to the wedge surface.

The so-called “shadow boundaries” are also shown in Fig 20. The Reflection-Shadow Boundary (RSB) is the angle beyond which no further reflections can take place for a given incoming angle. The Incident-Shadow Boundary (ISB) is that angle beyond which the wedge’s face blocks any incident rays from illuminating the observation point.

Keller derived the amplitude and phase terms by comparing the classical Geometric Optics (GO) solution with the exact mathematical solution calculated by Sommerfeld for a particular case where the boundary conditions were well known—an infinitely long, perfectly conducting wedge illuminated by a plane wave. Simply speaking, whatever was left over had to be diffraction terms. Keller combined these diffraction terms with GO terms to yield the total field everywhere.

Keller’s new theory became known as the Geometric Theory of Diffraction (abbreviated henceforth as GTD). The beauty of GTD was that in the regions where classical GO predicted zero fields, the GTD “filled in the blanks,” so to speak. For example, see **Fig 21**, showing the terrain for a hypothetical case, where a 60-foot high 4-element 15-meter Yagi illuminates a wide, perfectly flat piece of ground. A 10-foot high rock has been placed 400 feet away from the tower base in the direction of outgoing rays. **Fig 22** shows the elevation pattern predicted using reflection-only GO techniques. Due to blockage of the direct wave (A) trying to shoot past the 10-foot high rock, and due to blockage of (B) reflections from the flat ground in front of the rock, there is a “hole” in the smooth elevation pattern.

Now, doesn’t it defy common sense to imagine that a single 10-foot high rock will really have such an effect on a 15-meter signal? Keller’s GTD took diffraction effects into account to show that waves do indeed sneak past and over the rock to fill in the pattern. The whole GTD scheme is very clever indeed.

However, GTD wasn’t perfect. Keller’s GTD predicts some big spikes in the pattern, even though the overall shape of the elevation pattern is much closer to reality than a simple GO reflection analysis would indicate. The region right at the RSB and ISB shadow boundaries is where problems are found. The GO terms go to zero at these points because of blockage by the wedge, while Keller’s diffraction

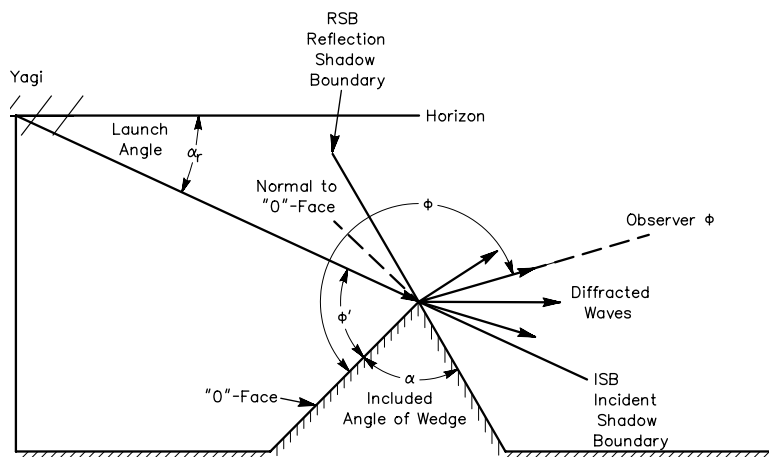


Fig 20—Diagram showing diffraction mechanism of ray launched at angle α_r below horizon at diffraction wedge, whose included angle is α . Referenced to the incident face (the “o-face” as it is called in UTD terminology), the incoming angle is ϕ' (phi prime). The wedge creates an infinite number of diffracted waves. Shown is one whose angle referenced to the o-face is ϕ , the so-called “observation angle” in UTD terminology.

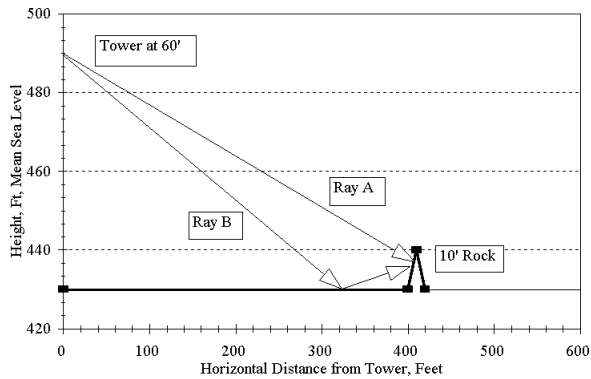


Fig 21—Hypothetical terrain exhibiting so-called “10-foot rock effect.” The terrain is flat from the tower base out to 400 feet, where a 10-foot high rock is placed. Note that this forms a diffraction wedge, but that it also blocks direct waves trying to shoot through it to the flat surface beyond, as shown by Ray A. Ray B reflects off the flat surface before it reaches the 10-foot rock, but it is blocked by the rock from proceeding further. A simple Geometric Optics (GO) analysis of this terrain without taking diffraction into account will result in the elevation response shown in Fig 22.

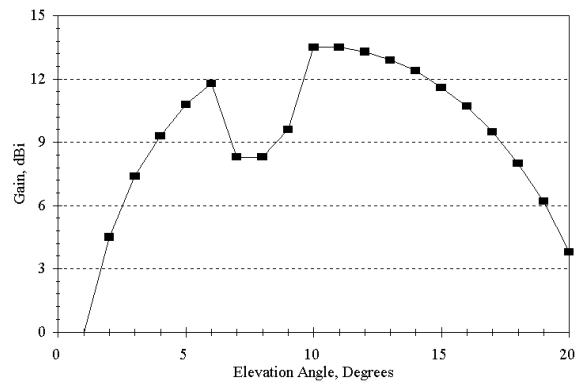


Fig 22—Elevation response for rays launched at terrain in Fig 21 from a height of 60 feet using a 4-element Yagi. This was computed using a simple Geometrical Optics (GO) reflection-only analysis. Note the “hole” in the response between 6° to 10° in elevation. It is not reasonable for a 10-foot high rock to create such a disturbance at 21 MHz!

terms tend to go to infinity at these very spots. In mathematical terms this is referred to as a “caustic problem.” Nevertheless, despite these nasty problems at the ISB and RSB, the GTD provided a remarkably better solution to diffraction problems than did classical GO.

In the early 1970s, a group at Ohio State University under R. G. Kouyoumjian and P. H. Pathak did some pivotal work to resolve this caustic problem, introducing what amounts to a clever “fudge factor” to compensate for the tendency of the diffraction terms at the shadow boundaries to go to infinity. They introduced what is known as a “transition function,” using a form of Fresnel integral. Most importantly, the Ohio State researchers also created several FORTRAN computer programs to compute the amplitude and phase of diffraction components. Now computer hackers could get to work!

The program that resulted is called *YT*, standing for “Yagi Terrain.” As the name suggests, *YT* analyzes the effect of local terrain—for Yagis only, and only for horizontally polarized Yagis. The accurate appraisal of the effect of terrain on vertically polarized signals is a far more complex problem than for horizontally polarized waves.

SIMULATION OF REALITY—SOME SIMPLE EXAMPLES FIRST

We want to focus first on some simple results, to show that the computations do make some sense by presenting some simulations over simple terrains. We’ve already described the “10-foot rock at 400 feet” situation, and showed where a simple GO reflection analysis is inadequate to the task without taking diffraction effects into account.

Now look at the simple case shown in **Fig 23**, where a very long, continuous downslope from the tower base is shown. Note that the scales used for the X and Y-axes are different: the Y-axis changes 300 feet in height (from 800 to 1100 feet), while the X-axis goes from 0 to 3000 feet. This exaggerates the apparent steepness of the downward slope, which is actually a rather gentle slope, at $\tan^{-1} (1000-850) / (3000 - 0) = -2.86^\circ$. In other words, the terrain falls 150 feet in height over a range of 3000 feet from the base of the tower.

Fig 24 shows the computed elevation response for this terrain profile, for a four-element horizontally polarized Yagi on a 60 foot tower. The response is compared to that of an identical Yagi placed 60

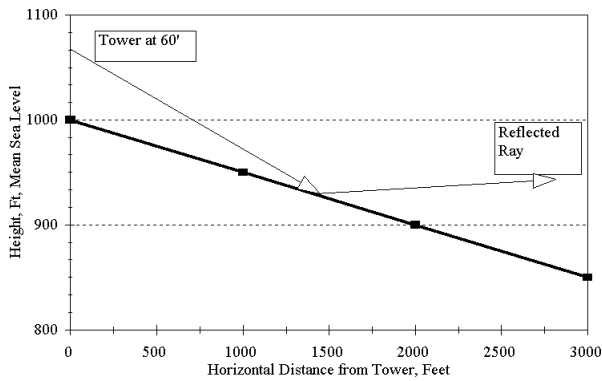


Fig 23—A long, gentle downward-sloping terrain. This terrain has no explicit diffraction points and can be analyzed using simple GO reflection techniques.

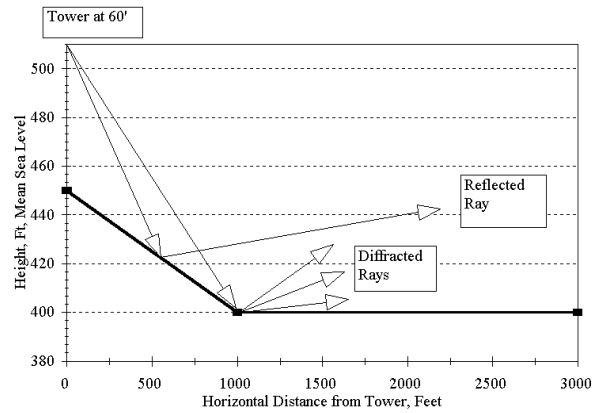


Fig 25—“Hill-Valley” terrain, with reflected and diffracted rays.

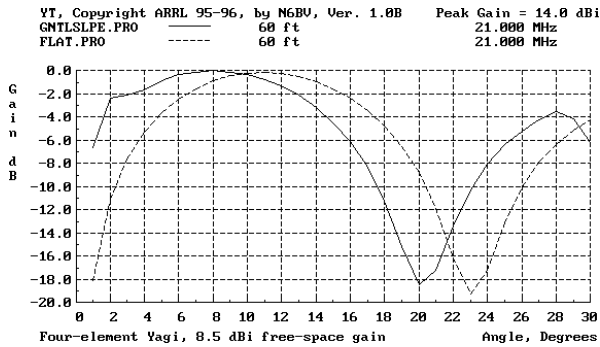


Fig 24—Elevation response for terrain shown in Fig 23, using a 4-element Yagi, 60-foot high. Note that the shape of the response is essentially shifted toward the left, toward lower elevation angles, by the angle of the sloping ground. For reference, the response for an identical Yagi placed over flat ground is also shown.

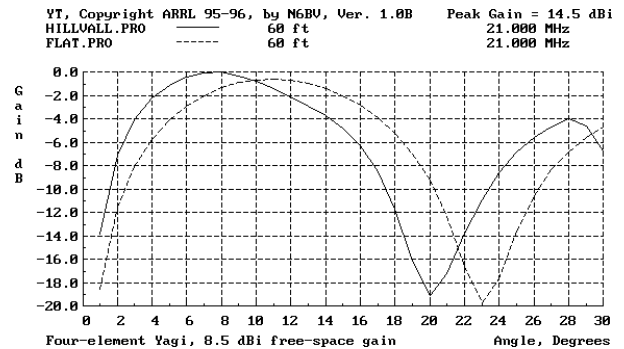


Fig 26—Elevation response computed by YT program for single 4-element Yagi at 60 feet above “Hill-Valley” terrain shown in Fig 25. Note that the slope has caused the response in general to be shifted toward lower elevation angles. At 5° elevation, the diffraction components add up to increase the gain slightly above the amount a GO-only analysis would indicate.

feet above flat ground. Compared to the “flatland” antenna, the hilltop antenna has an elevation response shifted over by almost 3° toward the lower elevation angles. In fact, this shift is directly due to the -2.86° slope of the hill. Reflections off the slope are tilted by the slope. In this situation there are no diffractions, just reflections.

Look at **Fig 25**, which shows another simple terrain profile, called a “Hill-Valley” scenario. Here, the 60-foot high tower stands on the edge of a gentle hill overlooking a long valley. Once again the slope of the hill is exaggerated by the different X and Y-axes. **Fig 26** shows the computed elevation response at 21.2 MHz for a 4-element Yagi on a 60-foot high tower at the edge of the slope.

Once again, the pattern is overlaid with that of an identical 60-foot-high Yagi over flat ground. Compared to the flatland antenna, the hilltop antenna’s response above 9° in elevation is shifted by almost 3° towards the lower elevation angles. Again, this is due to reflections off the downward slope. From 1° to 9°, the hilltop pattern is enhanced even more compared to the flatland antenna, this time by

diffraction occurring at the bottom of the hill.

Now let's see what happens when there is a hill ahead in the direction of interest. **Fig 27** depicts such a situation, labeled "Hill-Ahead." Here, at a height of 400 feet above mean sea level, the land is flat in front of the tower, out to a distance of 500 feet, where the hill begins. The hill then rises 100 feet over the range 500 to 1000 feet away from the tower base. After that, the terrain is a plateau, at a constant 500 feet elevation.

Fig 28 shows the computed elevation pattern for a 4-element Yagi 60-feet high on the tower, compared again with an overlay for an identical 60-foot high antenna over flat ground. The hill blocks low-angle waves directly radiated from the antenna from 0° to 2.3° . In addition, waves that would normally be reflected from the ground, and that would normally add in phase from about 2.3° to 12° , are blocked by the hill also. Thus the signal at 8° is down almost 5 dB from the signal over flat ground, all due to the effect of the hill. Diffracted waves start kicking in once the direct wave rises enough above the horizon to illuminate the top edge of the hill. These diffracted waves tend to augment elevation angles above about 12° , which reflected waves can't reach.

Is there any hope for someone in such a lousy QTH for DXing? **Fig 29** shows the elevation response for a truly heroic solution. This involves a stack of four 4-element Yagis, mounted at 120, 90, 60 and 30 feet on the tower. Now, the total gain is just about comparable to that from a single 4-element Yagi mounted over flat ground. Where there's a ham, there is a way!

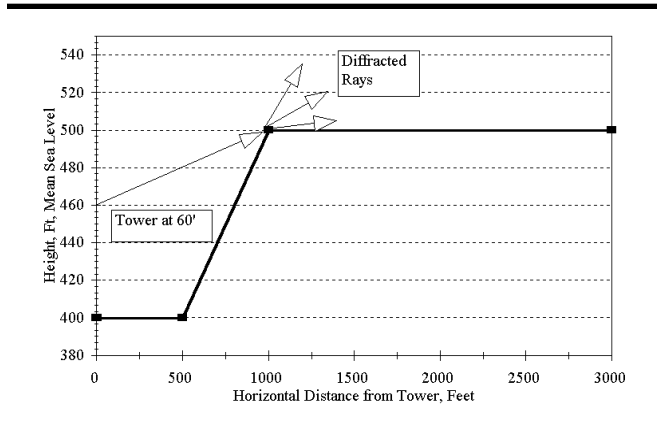


Fig 27—"Hill-Ahead" terrain, shown with diffracted rays created by illumination of the edge of the plateau at the top of the hill.

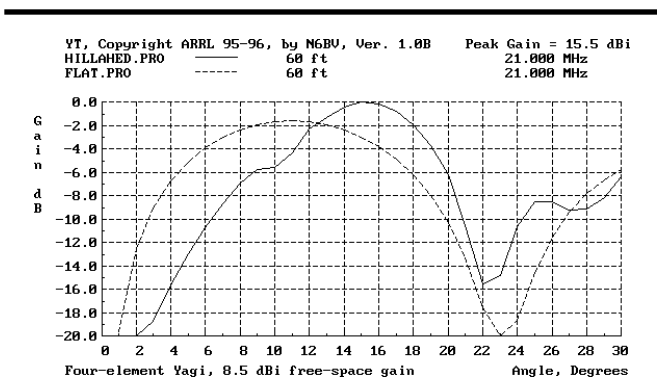


Fig 28—Elevation response computed by YT for "Hill-Ahead" terrain shown in Fig 27. Now the hill blocks direct rays and also precludes possibility of any constructive reflections. Above 10° , diffraction components add up together with direct rays to create the response shown.

At 5° elevation, four diffraction components add up (there are zero reflection components) to achieve the far-field pattern. This seems reasonable, because each of the four antennas is illuminating the diffraction point separately and we know that none of the four antennas can "see over" the hill directly to produce a reflection at a low launch angle.

You will note something new on Fig 29—another curve has appeared. The line with asterisks refers to the legend "W1-MA-EU.PRN." This curve portrays the relative percentage of time during which a particular elevation angle arrives in Massachusetts

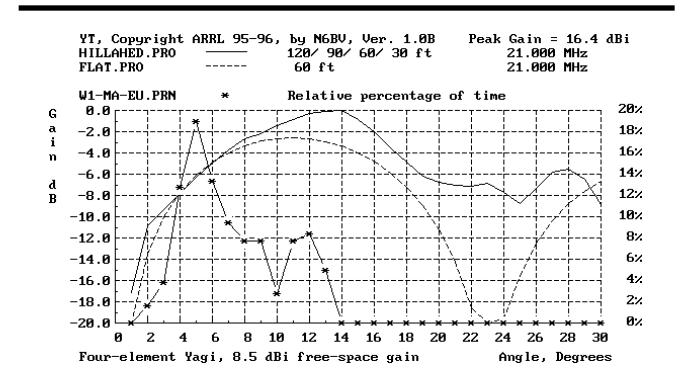


Fig 29—Elevation response of "heroic effort" to surmount the difficulties imposed by hill in Fig 27. This effort involves a stack of four 4-element Yagis in a stack starting at 120 feet and spaced at 30-foot increments on the tower. The response is roughly equivalent to a single 4-element Yagi at 60 feet above flat ground, hence the characterization as being a "heroic effort." Note that the elevation-angle statistics have been added to this plot as an overlay of asterisks.

from Europe. We have thus integrated on one graph the range of elevation angles necessary to communicate from New England to Europe (over the whole 11-year sunspot cycle) with the response attributed to the topography of a particular terrain.

For example, at an elevation angle of 5°, 15-meter signals arrive from Europe about 19% of the total number of times when the band is actually open. We can look at this another way. For about two-thirds of the times when the band is open on this path, the incoming angle is between 3° to 8°. For about one-quarter of the time, signals arrive above 10°, where the “heroic” four-stack is finally beginning to come into its own, sort of, anyway.

A More Complex Terrain

The results for simple terrains look reasonable; let’s try a more complicated real-world situation. **Fig 30** shows the terrain from the N6BV QTH toward Japan. The terrain is complex, with 17 different points *YT* identifies as diffraction points. **Fig 31** shows the *YT* output for three different types of antennas on 20 meters: a stack at 120 and 60 feet, the 120-foot antenna by itself, and then a 120-foot high antenna over flat ground, for reference. The elevation-angle statistics for New England to the Far East (Japan) are overlaid on the graph also, making for a very complicated looking picture—it is a *lot* easier to decipher the lines on the color CRT, by the way than on a black-and-white printer.

Examination of the detailed data output from *YT* shows that at an elevation angle of 5°, the peak percentage angle (19% of the time when the band is open), there are three reflection components for the 120/60-foot stack, but there are also 25 diffraction components! There are many, many signals bouncing around off the terrain on their trip to Japan. Note that because of blockage of some parts of the terrain, the 60-foot high Yagi cannot illuminate all the diffraction points, while the higher 120-foot Yagi is able to “see” these diffraction points.

It is fascinating to reflect on the thought that received signals coming down from the ionosphere to the receiver are having encounters with the terrain, but from the opposite direction. It’s not surprising, given these kinds of interactions, that transmitting and receiving might not be totally reciprocal.

It is interesting that the 120/60-foot stack, indicated by the light solid line in Fig 31, achieves its peak gain of 18.4 dBi at 8° elevation, where it is about equal to the single 120-foot high 4-element Yagi. At 11° elevation, the difference is about 7 dB in favor of the stack. Numerous times such a marked difference in performance between the stack and each antenna by itself have been observed. Such performance differences due to complex terrain may in fact partly account for why stacks often seem to be “magic” compared to single Yagis at comparable heights.

Certainly there is no way a two-beam stack can

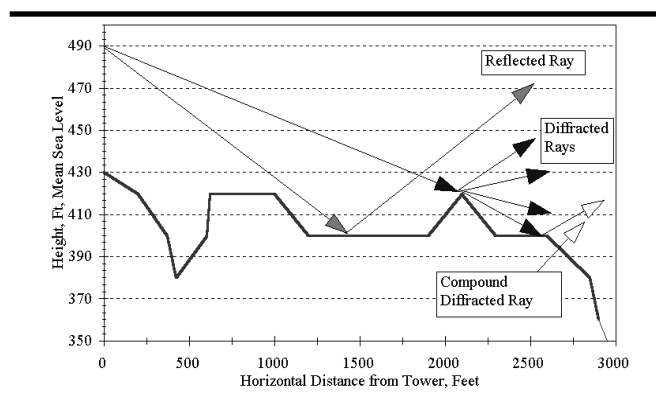


Fig 30—Terrain of N6BV in Windham, New Hampshire, toward Japan. *YT* identifies 17 different points where diffraction can occur.

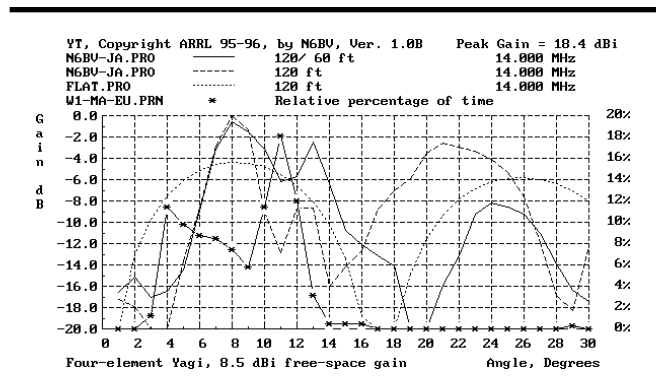


Fig 31—Elevation responses computed by *YT* for N6BV terrain shown in Fig 30, for a stack of two 4-element Yagis at 120 and 60 feet, together with the response for a single Yagi at 60 feet. The response due to many diffraction and reflection components is quite complicated! The response for a single 4-element Yagi over flat ground is shown by the light dotted line, for reference.

actually achieve a 7 dB difference in gain over a single antenna due to stacking alone. Computer modeling over flat ground indicates a maximum practical gain difference on the order of 2.5 to 3 dB, depending on the spacing and interaction between individual Yagis in a stack of two—the uneven terrain is giving the additional focusing gain. Note that you still don't get something for nothing. While gain at particular angles may be enhanced by terrain focusing, gain at other angles is degraded compared to a flat-ground terrain.

Much of the time when comparisons are being made, the small differences in signal are difficult to measure meaningfully, especially when the QSB varies signals by 20 dB or so during a typical QSO.

USING YT Generating a Terrain Profile

The program uses two distinct algorithms to generate the far-field elevation pattern. The first is a simple reflection-only Geometric Optics (GO) algorithm. The second is the diffraction algorithm using the Uniform Theory of Diffraction (UTD). These algorithms work with a digitized representation of the terrain profile for a single azimuthal direction—for example, toward Japan or toward Europe.

The terrain file is generated manually using a topographic map and a ruler or a pair of dividers. The YT.TXT file on the accompanying diskette gives complete instructions on how to create a terrain file. The process is simple for people in the USA. Mark on the US Geological Survey 7.5 minute map the exact location of your tower. You will find 7.5 minute maps available from some local sources, such as large hardware stores, but the main contact point is the U.S. Geological Survey, Denver, CO 80225 or Reston, VA 22092. Call 1-800-MAPS-USA. Ask for the folder describing the topographic maps available for your geographic area. Many countries outside the USA have topographic charts also. Most are calibrated in meters, however. To use these with *TA*, you will have to convert meters to feet by multiplying meters by 3.28.

Mark off a pencil line from the tower base, in the azimuthal direction of interest, perhaps 45° from New England to Europe, or 335° to Japan. Then measure the distance from the tower base to each height contour crossed by the pencil line. Enter the data at each distance/height into an ASCII computer file, whose filename extension is “PRO,” standing for “profile.”

Fig 32 shows a portion of the USGS map for the N6BV QTH in Windham, NH, along with lines scribed in several directions towards various parts of Europe and the Far East. Note that the elevation heights of the intermediate contour lines are labeled manually in pencil in order to make sense of things. It is very easy to get confused unless you do this!

The terrain model used by *YT* assumes that the terrain is represented by flat “plates” connecting the elevation points in the *.PRO file with straight lines. The model is two dimensional, meaning that range and elevation are the only data for a particular azimuth. In effect, *YT* assumes that the width of a terrain plate is wide relative to its length. Obviously, the world is three dimensional. If your shot in a particular direction involves aiming your Yagi down a canyon with steep walls, then it's pretty likely that your actual elevation pattern will be different than what *YT* tells you. The signals must careen horizontally from wall to wall, in addition to being affected by the height changes of the terrain. *YT* isn't designed to do canyons.

To get a true 3-D picture of the full effects of terrain, a terrain model would have to show azimuth, along with range and elevation, point-by-point for about a mile in every direction around the base of the tower. After you go through the pain of manually creating a profile for a single azimuth, you'll appreciate the immensity of the process if you try to create a full 360° 3-D profile.

Digital terrain maps are available in some locations. However, be cautioned that the digitized data from such databases is fairly crude in resolution. No doubt, the data is adequate to keep a Cruise Missile flying above the terrain, one of the original intents for digitized terrain data. The data is probably adequate for many other non-military purposes too. But it is rarely sufficiently detailed to be truly representative of what your antenna looks down at from the tower.

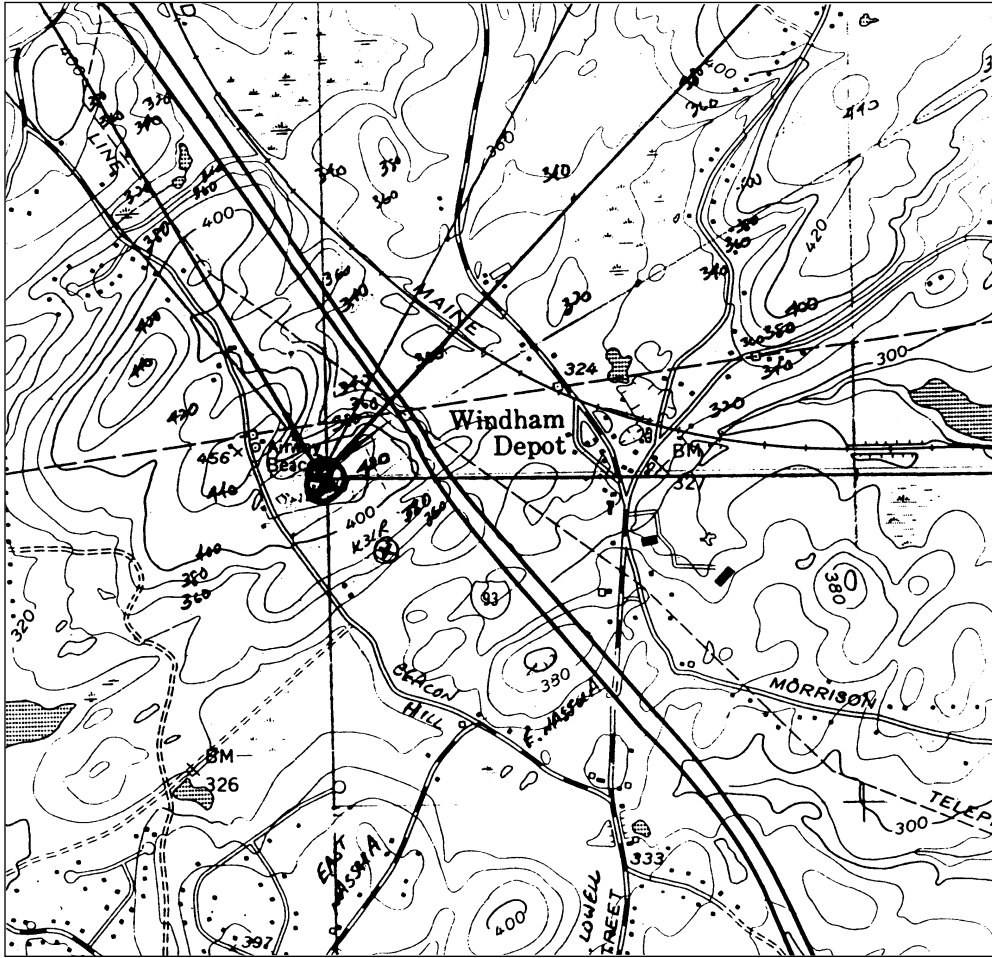


Fig 32—A portion of USGS 7.5 minute topographic map, showing N6BV QTH, together with marks in direction of Europe and Japan from tower base. Note that the elevation contours were marked by hand to help eliminate confusion. This required a magnifying glass and a steady hand!

Algorithm for Ray-Tracing the Terrain

There are a number of mechanisms that should be taken into account as a ray travels over the terrain:

1. Classical ray reflection, with Fresnel ground coefficients.
2. Direct diffraction, where a diffraction point is illuminated directly by an antenna, with no intervening terrain features blocking the direct illumination.
3. When a diffracted ray is subsequently reflected off the terrain.
4. When a reflected ray encounters a diffraction point and causes another series of diffracted rays to be generated.
5. When a diffracted ray hits another diffraction point, generating another whole series of diffractions.

Certain unusual, bowl-shaped terrain profiles, with sheer vertical faces, can conceivably cause signals to reflect or diffract in a backward direction, only to be reflected back again in the forward direction by the sheer-walled terrain to the rear. *YT* does not accommodate these interactions, mainly because to do so would increase the computation time too much.

YT's Internal Antenna Model

The Yagi antenna used inside *YT* can be selected by the operator to be anywhere from a 2-element to an 8-element Yagi. The default assumes a simple cosine-squared response equivalent to a 4-element Yagi in free space. *YT* traces rays only in the forward direction from the tower along the azimuth of interest. This keeps the algorithms reasonably simple and saves computing time, while minimizing memory requirements. Since the Yagi model assumes that the antenna has a decent front-to-back ratio,

there is no need to worry about signals bouncing off the terrain behind the tower, something that would be necessary for a dipole, for example.

YT considers each Yagi in a stack as a separate point source. The simulation begins to fall apart if a traveling wave type of antenna like a rhombic is used, particularly if the terrain changes under the antenna—that is, the ground is not flat under the entire antenna. For a typical Yagi, even a long-boom one, the point-source assumption is reasonable. The internal antenna model also assumes that the Yagi is horizontally polarized. *YT* does not do vertically polarized antennas.

YT compares well with the measurements for the horizontal antennas described earlier by Jim Breakall, WA3FET, using a helicopter in Utah. Breakall's measurements were done with a 15-foot high horizontal dipole.

More Details About YT

Frequency Coverage

YT can be used on frequencies higher than the HF bands, although the graphical resolution is only a degree. The patterns above about 100 MHz thus look rather grainy. The UTD is a “high-frequency asymptotic” solution, so in theory the results get more realistic as the frequency is raised. Keep in mind too that *YT* is designed to model launch angles for skywave propagation modes, including F-layer and even sporadic E. Since by definition the ionospheric launch angles include only those above the horizon, direct line-of-sight UHF modes involving negative launch angles are not considered in *YT*.

See YT.TXT for further details on the operation of the *YT* program. This file, as well as sample terrain profiles for “big-gun” stations, is located on the disk accompanying this book.

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The last major study that appeared in the amateur literature on the subject of local terrain as it affects DX appeared in four *QST* “How's DX?” columns, by Clarke Greene, K1JX, from October 1980 to January 1981. Greene's work was an update of a landmark September 1966 *QST* article entitled “Station Design for DX,” by Paul Rockwell, W3AFM. The long-range profiles of several prominent, indeed legendary, stations in Rockwell's article are fascinating: W3CRA, W4KFC and W6AM.